Semiparametric Identification of Multidimensional Screening Models

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The purpose of this paper:

To identify underlying functions even in the presence of unobserved endogenous variables

(1) Finds such a condition

(2) Suggests a method

Example: Monopoly in the cable television service market

The service the monopolist provides has two features:
(1) the number of channels
(2) the average quality of channels
that each household subscribes.

The problem is that:
(2) is not observable (or hard to evaluate).
(2) is endogenous ($\because$ decided by each household).

The existing method cannot solve this problem:
We cannot identify preferences and cost functions.

For example, let the utility be

$$U(x, \varepsilon, z, \xi) = x_i' A z_k + \xi_k + \varepsilon_{ik} - p_k$$

where

- x_i: characteristics of consumer i
- z_k: the number of channels in alternative k
- ξ_k: the average quality of alternative k
- ε_{ik}: taste consumer i has for k
- p_k: the price of alternative k

We want to estimate A from

$$U(x, \varepsilon, z, \xi) = x_i' A z_k + \xi_k + \varepsilon_{ik} - p_k$$

The literature regards ξ_k as exogenous and assumes that

$$E[\xi_k | z_k] = 0$$

If this assumption is reasonable, we can take some procedures to estimate A as
(1) obtain market shares from observable data,
(2) solve for ξ_k that is consistent with market shares, and
(3) use GMM with the above moment condition.

However, the above assumption is invalid when ξ_k (and z_k as well) is endogenous.

The idea to overcome this problem is

to use the bunching region where z and ξ are perfectly tied up with each other.

Definition (bunching):

Offering a limited set of goods that different consumers (in terms of characteristics) will choose to buy.

Underlying theory

The monopolist wants to maximize profit.
First-degree price discrimination is not feasible!
$\rightarrow$ Wants to build a product line that maximizes profit
subject to
(1) IR constraints and
(2) IC constraints.

The result: There is optimal non-linear pricing such that
(1) z and ξ are monotonely increasing in x and ε.
(2) price is montonely increasing in z and ξ.

There could be a region of x and ε on which z and ξ are constant $\rightarrow$ bunching!

In the bunching region, all the product attributes are uniquely tied, so the price does not change when the characteristics of consumers change.

Use this to find equations that enable identification.

Once we recover preferences and cost functions, we can
(1) anticipate the result of introduction of a new good,
(2) measure its effect on welfare more accurately.

A drawback of this method is that it can be used only when the market is a monopoly.

Model

Consumers have utility function

$$\begin{aligned} U(x, \varepsilon, z, \xi) &= \phi(\theta_1 z + \xi)x + (\theta_2 z + \xi)\varepsilon - p(z, \xi) \\ &= \phi(\theta_1 z + \xi)x + \theta_2 \varepsilon z + \varepsilon \xi - p(z, \xi) \end{aligned}$$

where ϕ is a monotonic function.
Consumers choose z and ξ that maximize their utility.

The monopolist has cost function

$$C(z, \xi) = c(z) + \gamma \xi^2$$

where c is a convex function.
It decides which pairs of (z, ξ) to provide at what $p(z, \xi)$.

Consumers' problem:

$$V(x,\varepsilon) = \max_{(z,\xi)\in Q} U(x,\varepsilon,z,\xi) \qquad \text{subject to } V(x,\varepsilon) \geq \overline{V}$$

where Q is a set of (z,ξ) provided by the monopolist.

Envelope theorem yields

$$V_x(x,\varepsilon) = \phi(\theta_1 z + \xi)$$
$$V_\varepsilon(x,\varepsilon) = \theta_2 z + \xi$$

Rearranging

$$\begin{bmatrix} z \\ \xi \end{bmatrix} = \begin{bmatrix} \theta_1 & 1 \\ \theta_2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \phi^{-1}\Big(V_x(x,\varepsilon)\Big) \\ V_\varepsilon(x,\varepsilon) \end{bmatrix} \tag{1}$$

Now with this implemetation result, maximize profit.

Note that profit and consumers' surplus add up to social surplus. This yields

$$\pi + V(x,\varepsilon) = U(x,\varepsilon,z,\xi) + p(z,\xi) - C(z,\xi)$$

or

$$\pi = \phi(\theta_1 z + \xi)x + (\theta_2 z + \xi)\varepsilon - C(z,\xi) - V(x,\varepsilon)$$

Use (1) to rewrite as

$$\begin{aligned}\pi &= xV_x + \varepsilon V_\varepsilon - c(z) - \gamma\xi^2 - V \\ &= xV_x + \varepsilon V_\varepsilon - c\left(\frac{\phi^{-1}(V_x) - V_\varepsilon}{\theta_1 - \theta_2}\right) - \gamma\left(\frac{\theta_1 V_\varepsilon - \theta_2\phi^{-1}(V_x)}{\theta_1 - \theta_2}\right)^2 - V\end{aligned}$$

which is a function of x and ε only.

The monopolist's problem:

$$\Phi(V) = \max_{V} \int_{(x,\varepsilon)\in\Omega} \pi(x,\varepsilon)dF(x,\varepsilon)$$

subject to $V(x,\varepsilon) \geq \overline{V}$

and $V(x,\varepsilon)$ is convex.

where Ω is a whole set of (x,ε).

There are two characterization results.

Proposition 1 omitted.

Proposition 2 characterzes (x, ε, z, ξ) in each bunch, which is a straight line when utility is quasilinear.

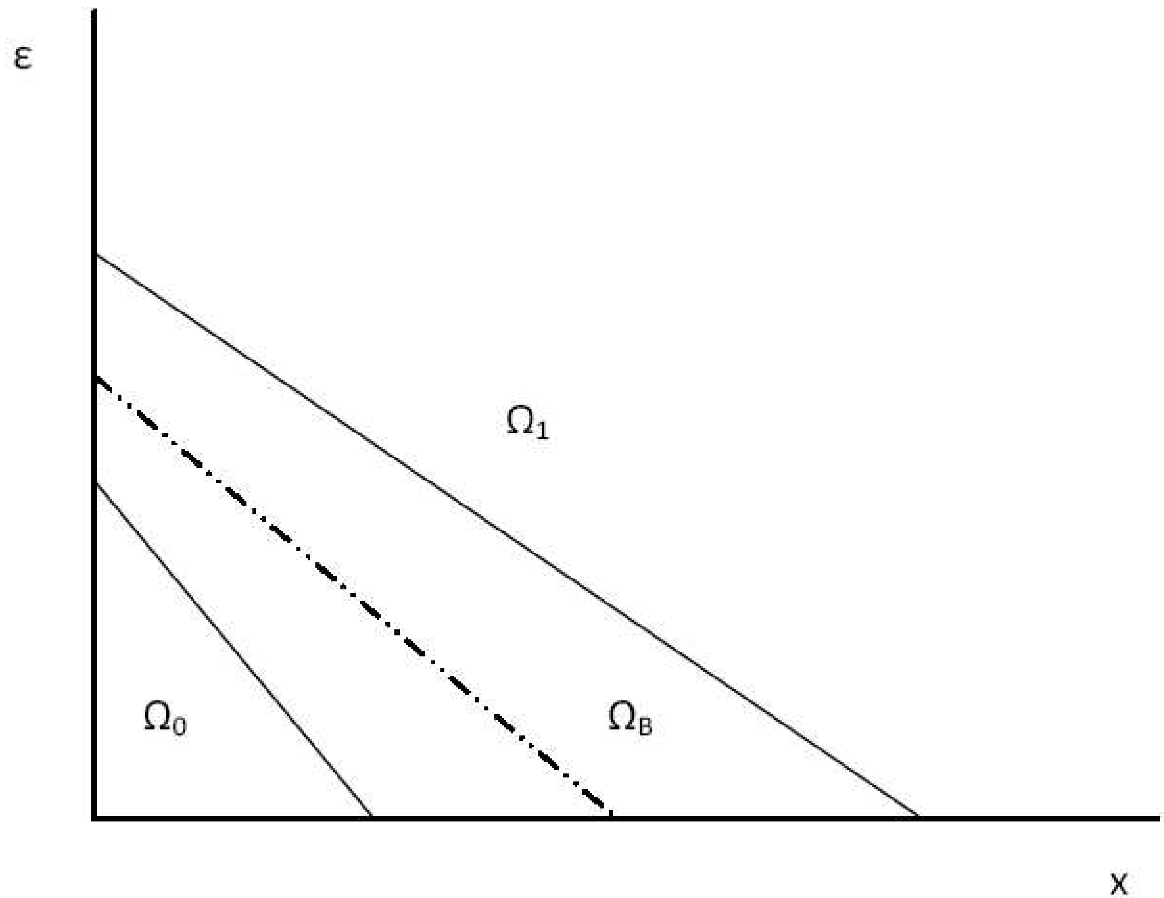

Figure 3: Division of the set of consumers in the presence of bunching.

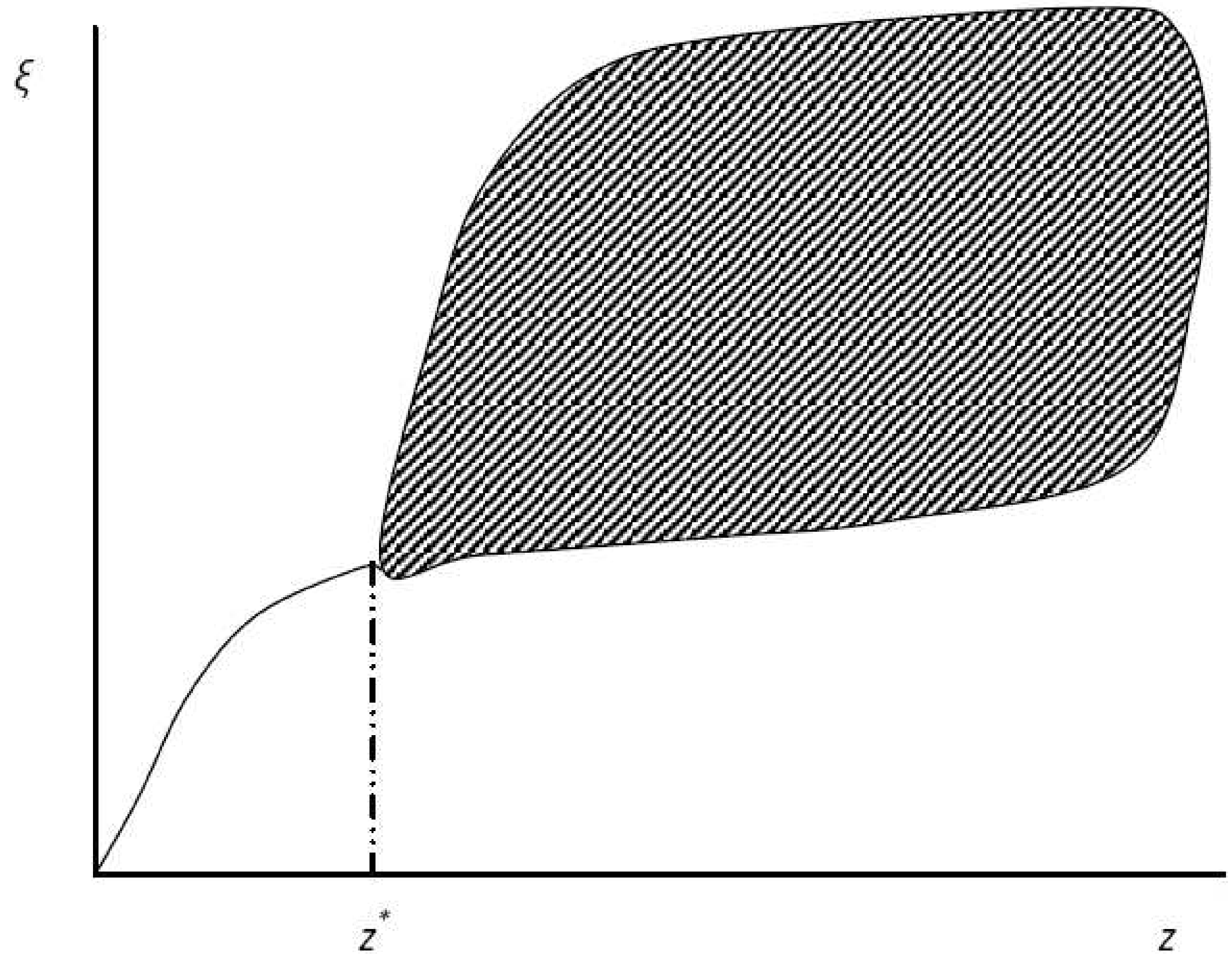

Figure 4: Product-line offered by the monopolist.

Identification

Proposition 3 The bunching region is determined by the points where

$$\frac{\partial}{\partial x}p(x,z) = 0$$

Sketch of Proof

In the bunching region, $\xi = g(z)$, so

$$p(z,\xi) = p\Big(z, g(z)\Big) \ : \ \text{function of } z \text{ only}$$

In the other region,

$$p = p\Big(z, s(x,\varepsilon)\Big) = p\Big(z, s\Big(x, m^{-1}(x,z)\Big)\Big)$$

and thus $\frac{\partial p}{\partial x} \neq 0$.

Recall each bunch (z, ξ) is characterized by a straight line

$$\varepsilon + \beta(z)x = \tau(z)$$

In the bunching region,

$$\begin{aligned} U &= \phi(\theta_1 z + \xi)x + (\theta_2 z + \xi)\varepsilon - p(z, \xi) \\ &= \phi\Big(\theta_1 z + g(z)\Big)x + \Big(\theta_2 z + g(z)\Big)\varepsilon - p\Big(z, g(z)\Big) \end{aligned}$$

Consumers' FOC in the bunching region is

$$\frac{\partial U}{\partial z} = 0$$

which yields

$$\begin{aligned} \beta(z) &= \phi'\Big(\theta_1 z + g(z)\Big)\frac{\theta_1 + g'(z)}{\theta_2 + g'(z)} \\ \tau(z) &= \frac{p'(z)}{\theta_2 + g'(z)} \end{aligned}$$

In the bunching region, given $X = x$ and $Z = z$, every consumer with

$$\varepsilon \leq \tau(z) - \beta(z)x$$

would buy at most z and corresponding ξ.

Propensity of consumers with x to buy at most z is

$$\begin{aligned} P(z,x) &\equiv F_{Z|X=x}(z) \\ &= \Pr(Z \leq z|X = x) \\ &= \Pr\left(\varepsilon \leq \tau(z) - \beta(z)x \middle| X = x\right) \\ &= F_{\varepsilon}\Big(\tau(z) - \beta(z)x\Big) \end{aligned}$$

since $\varepsilon \perp x$ is assumed.

Use $P(z,x) = F_\varepsilon\Big(\tau(z) - \beta(z)x\Big)$ to identify $\tau(z)$ and $\beta(z)$.

For example, for $\beta(z)$,

$$\begin{aligned} P_z &= F'_\varepsilon(\cdot)\Big(\tau'(z) - \beta'(z)x\Big) \\ P_x &= F'_\varepsilon(\cdot)\Big(-\beta(z)\Big) \end{aligned}$$

So

$$\frac{P_z}{P_x} = \frac{\beta'(z)x - \tau'(z)}{\beta(z)}$$

Differentiate w.r.t. x and integrate over z, then

$$\int_0^s \frac{\partial}{\partial x}\left(\frac{P_z}{P_x}\right) dz = \log\beta(s) - \log\beta(0)$$

Normalizing $\beta(0) = 1$ yields $\beta(s)$.

Proposition 4 identifies θ_1,θ_2,γ and $c(z)$.

Proposition 5 identifies F_ε up to a normalization.

Benchmark case

Berry, Levinsohn and Pakes (1995) assumes $E[\xi|z] = 0$, but we have seen that

$$E[\xi|z] = g(z)$$

which is increasing in z when ξ is endogenous.