Econ 201A Microeconomic Theory Fall 2001

Time allowed: Three (30) hours plus ten minutes for reading only. Answer four (4) questions only. You must derive your answers. A correct statement with no explanation or derivation will not score points.

1. Production and cost

A firm produces a single output q and is a price taker in all input markets.

(a) Provide conditions under which a firm will increase its output if the cost of the jth input falls.

(b) Are these conditions satisfied if $q = (\sum_{j=1}^{n} z_j^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ $\sigma > 0$, $\sigma \neq 1$?

(c) Returning to a general production function $q = F(z)$, under what conditions is the cost function $C(q,r)$ a concave function of r?

(d) Under what conditions is $C(q,r)$ a concave function of q?

2. Risk averse and risk neutral consumers

Some of the consumers in the economy are risk neutral and others are risk averse. There are two state of nature and the probability of state s is π_s. Each risk averse consumer has VNM utility function $U(c) = \pi_1 c_1^{1/2} + \pi_2 c_2^{1/2}$. The aggregate endowment in the economy is $\omega = (a, 2a)$.

(a) Explain why it is possible to analyze the equilibrium and efficient allocations in this economy by analyzing a 2 person economy.

(b) Under what conditions, if any, will the Walrasian equilibrium price ratio p_1 / p_2 be equal to the odds (ratio of probabilities)?

(c) Under what circumstances (if any) will the equilibrium price ratio ever be lower than the odds?

(d) Determine the range of possible equilibrium price ratios in this economy.

3. Time constrained choice

An individual works for a fixed number of hours, T, out of each 24 hour day. There are n commodities $x = (x_1,..., x_n)$ and the price vector is $p = (p_1,..., p_n)$. Each unit of commodity j, $j = 1,....,n$ takes t_j units of time to consume. The hourly wage rate is w.

(a) Write down the consumer's optimization problem and first order conditions if the individual has a continuously differentiable strictly increasing utility function $U(x_1,..., x_n)$.

(b) Under what conditions (if any) will one or other of the constraints be non-binding?

(c) Obtain and interpret an expression for the individual's marginal rate of substitution at the optimum.

(d) Next let the number of hours worked be a choice variable. Re-solve the problem and again obtain an expression for $MRS(x_i, x_j)$. Interpret this expression.

(e) What difference would it make if pure leisure (lying round and consuming nothing.) were to be added to the list of commodities?

4. **Input and output prices**

A firm has a production function $q = F(z) = A(\frac{z_1}{\alpha_1})^{\alpha_1}(\frac{z_2}{\alpha_2})^{\alpha_2}(\frac{z_3}{\alpha_3})^{\alpha_3}$, $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

The input price vector is r.

(a) Show that $\frac{\partial q}{\partial z_j} = \frac{\alpha_j q}{z_j}$.

(b) Form the Lagrangian in order to solve for the cost minimizing choice of inputs.

(c) Interpret the Lagrange multiplier.

(d) From the first order conditions, show that $\frac{z_j}{\alpha_j} = \frac{\lambda q}{r_j}$.

Hence solve for the Lagrange multiplier.

(e) What is the marginal and average cost of the firm?

(f) If the firm is a price taker, in all markets, what can you say about the Walrasian equilibrium output price p?

5. **Optimal Investment**

A firm with an output capacity q_t in period t can sell q units at a price $p_t = a - bq$. Capacity depreciates at the rate θ. With a new investment of x_t, capacity next period is $q_{t+1} = (1-\theta)q_t + x_t$. The cost of adding x_t units of capacity is $C(x_t) = \frac{1}{2}x_t^2$. The interest rate is r and the firm's objective is to maximize the present value of the profit stream.

(a) Write down the optimization problem given that there are T periods and that capacity in period T+1 must be at least $\bar{q}_{T+1}$.

(b) Obtain first order conditions for the solution of the optimization problem and hence show that along one phase boundary $MR(q_t) = (r+\theta)x_t$

(c) Given an infinite horizon, characterize as completely as you can the optimal path of output and prices, starting from a small initial capacity.

(d) Solve for the long run output of the firm.

(e) An unanticipated technological change in period S reduces the cost of new investment to $C(x_t) = \beta x_t$, where β is small. What is the output path for $t > S$?

6. State Claims equilibrium

There are two periods, and two states in period 2. There is one commodity. All individuals have the same VNM utility $V(c_1, c_{2s}) = \ln c_1 + \delta \ln c_{2s}$, $s = 1, 2$. The probabilities of the states are $\pi_1 = \frac{1}{4}$, $\pi_2 = \frac{3}{4}$. The period 1 endowment is 100. The period 2 endowment is $(\omega_{21}, \omega_{22}) = (200, 50)$, where ω_{2s} is the endowment in period 2 if the state is *s*.

(a) Solve for the Walrasian equilibrium state claims prices if the spot price of the commodity is 1.

Suppose, henceforth, that there is a technology which can transform *z* units of the period 1 commodity into $q_s(z) = z\alpha_s$ units in period 2 if the state is *s*.

(b) For the special case of costless storage ($\alpha_1 = \alpha_2 = 1$), for what values of the discount factor will there be no storage in the Walrasian equilibrium?

(c) For what values of the discount factor is it not Pareto-efficient to store?

(d) Suppose $\alpha_1 = 2$ and $\alpha_2 = 1$. For what values of the discount factor is there no production in the Walrasian equilibrium?

Good Luck!