ECON 271B homework #1

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(1) OLS assumes that $E[\omega_{it}|k_{it}, l_{it}] = 0$. The OLS estimates of the coefficients are

coefficient	estimate	standard error
β_0	2.8176	0.0768
β_1	0.3935	0.0135
β_2	1.0969	0.0291

Using Δ-method,

$$\widehat{\beta}_1 + \widehat{\beta}_2 = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \widehat{\beta} \dot{\sim} N(\beta_1 + \beta_2, V)$$

where

$$V = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} Var\left(\widehat{\beta}\right) \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

So the returns to scale estimate is $\widehat{\beta}_1 + \widehat{\beta}_2 = 1.4904$, and its standard error is $\sqrt{V} = 0.0203$. Here the estimates are biased since the assumption that $E[\omega_{it}|k_{it}, l_{it}] = 0$ is not reasonable. Firms are likely to choose more k_{it} and l_{it} if they observe higher ω_{it}. So ω_{it} has a positive correlation with k_{it} and l_{it}. In the classical linear regression model,

$$E\left[\widehat{\beta}\right] = E\left[(X'X)^{-1}X'y\right] = \beta + E\left[(X'X)^{-1}X'\varepsilon\right]$$

and thus if ε and X are positively correlated, the last term is positive, which means that $\widehat{\beta}$ is biased upward. So in the question, the OLS estimates are expected to be biased upward.

(2) In the plant fixed effects model, we use 1075 dummy variables to estimate β_1 and β_2. The estimates of the coefficients are

coefficient	estimate	standard error
β_1	0.1872	0.0494
β_2	0.4200	0.0518

These estimates are quite smaller than the OLS estimates. This result is consistent with the analysis in (1), but the returns to scale does not seem reasonable. Standard errors are larger than under OLS. This might result from smaller degrees of freedom or measurement error. The key assumption of this model is that ω_{it} is constant over time for every firm i. In other words, the model is

$$y_{it} = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} + \omega_i + \varepsilon_{it}$$

This assumption seems too strong.

(3) At the 1st stage of LP, we can obtain the estimate of β_2 as

coefficient	estimate	standard error
β_2	0.6302	0.0297

We use the moment condition

$$E\begin{bmatrix} \omega_{it} \\ \xi_{it}k_{it} \end{bmatrix} = 0$$

The second condition comes from the assumption that $E[\xi_{it}|k_{it}] = 0$. This depends on the timing assumption that k_{it} is chosen at period $t-1$, which means that k_{it} is chosen before the new innovation ξ_{it} is realized.

(4) The LP procedure yields the following result.

coefficient	estimate	standard error
β_0	2.7445	0.9280
β_1	0.5259	0.1085

(5) In the ACF model, β_2 cannot be identified in the 1st stage. Since $g^{-1}(k_{it}, l_{it}, m_{it})$ has 1st order polynomial in l_{it}, we can just identify $\gamma_2 = \beta_2 + g_2$. We use the moment condition

$$E\begin{bmatrix} \omega_{it} \\ \xi_{it}k_{it} \\ \xi_{it}l_{it} \end{bmatrix} = 0$$

The last condition comes from the assumption that $E[\xi_{it}|l_{it}] = 0$. This means that l_{it} should be in the information set at period $t-1$. In other words, l_{it} should be chosen before the new innovation ξ_{it} is realized. If we use $E[\xi_{it}l_{it-1}] = 0$ instead, this would not assume that l_{it} is chosen before ξ_{it} is realized, so this is the weaker assumption than that used in the moment condition. Note that l_{it-1} is always in the information set at period $t-1$. So if the assumption that $E[\xi_{it}|I_{it-1}] = 0$ is reasonable, we can always use $E[\xi_{it}l_{it-1}] = 0$. The ACF procedure estimates β as follows.

coefficient	estimate	standard error
β_0	2.8677	0.8960
β_1	0.4553	0.1207
β_2	0.8420	0.0683

Compared to the estimates of LP procedure, the estimate of β_1 is less, but that of β_2 is greater. Under the LP assumption, β_2 is not correctly identified. It is underestimated in the LP procedure.