Coordination Games, Multiple Equilibria and the Timing of Radio Commercials

Andrew Sweeting

presented by Yang, Yong Hyeon

- Multiple Equilibria usually cause a problem.

(1) Which equilibrium will be played?

∗ strategies, priors, timing, ...

(2) How do we estimate parameters?

∗ We may have two equations of parameters.

- In this paper, multiple equilibria help with identification.
  - We have one equation.
  - How can we estimate many parameters with one equation?
  - Multiple equilibria give us a number of equations.

- The model
  - Players choose one of two actions $t \in \{0, 1\}$

$$\pi_{it} = \beta_t + \alpha P_{-it} + \varepsilon_{it}$$

where
π : profit
β_t : common factor affecting profit
P_{-it} : proportion of other players choosing action t
ε_{it} : individual error (iid)

  - i chooses action 1 when $\pi_{i1} \geq \pi_{i0}$

- Bayesian Nash Equilibrium

  - ε_{it} is observed by i, but not by the others.

  - i chooses action 1 when

$$\beta_1 + \alpha E[P_{-i1}|S_{-i}] + \varepsilon_{i1} \geq \beta_0 + \alpha E[P_{-i0}|S_{-i}] + \varepsilon_{i0}$$

  - Assume iid logit error for ε_{it}, then

$$\Pr(\pi_{i1} \geq \pi_{i0}) = \frac{e^{\beta_1 + \alpha E[P_{-i1}|S_{-i}]}}{e^{\beta_1 + \alpha E[P_{-i1}|S_{-i}]} + e^{\beta_0 + \alpha E[P_{-i0}|S_{-i}]}}$$

  - A priori, i chooses action 1 with probability $\Pr(\pi_{i1} \geq \pi_{i0})$
    : i's mixed strategy

- Normalize $\beta_0 = 0$
  - Cannot identify β_0 and β_1 separately.

- Symmetric Equilibrium
  - Everybody uses the same strategy

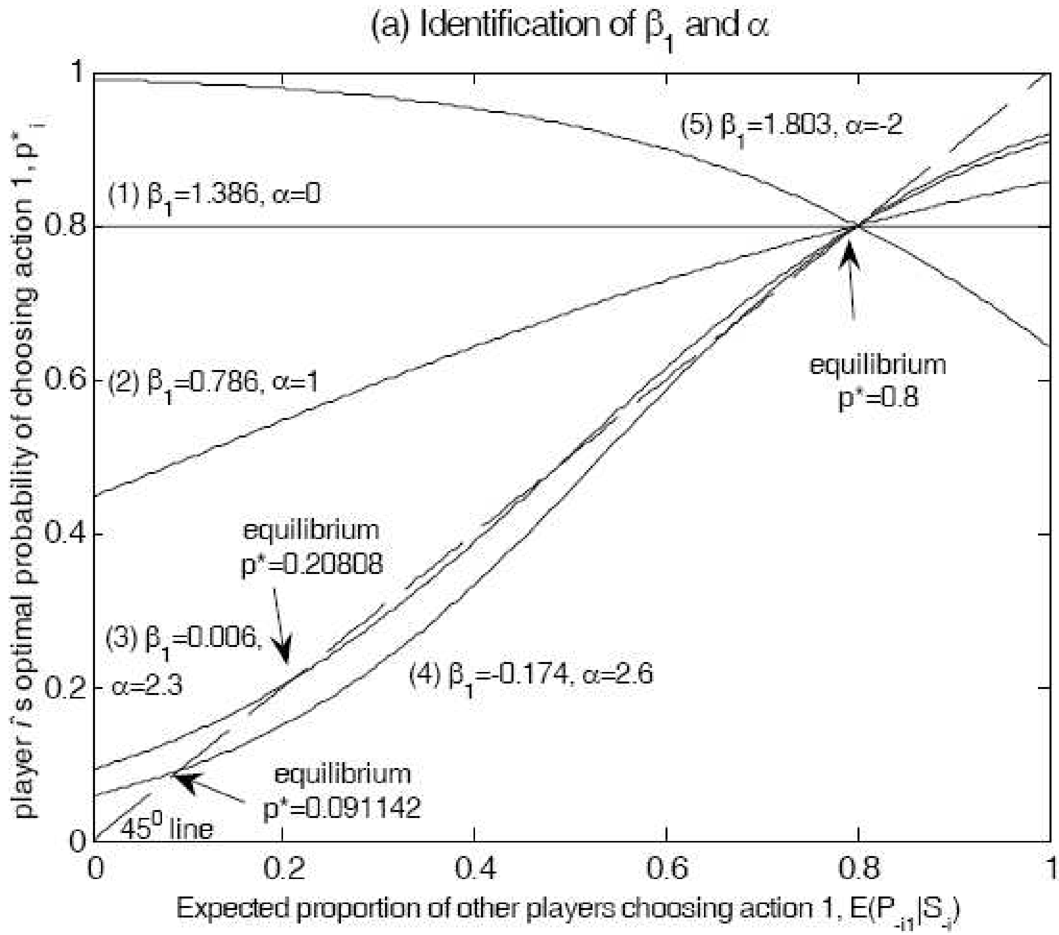
$$\Pr(\pi_{i1} \geq \pi_{i0}) = E[P_{-i1}|S_{-i}] \equiv p^*$$

  - The equation becomes

$$p^* = \frac{e^{\beta_1 + \alpha p^*}}{e^{\beta_1 + \alpha p^*} + e^{\alpha(1-p^*)}}$$

- Identification of parameters
  - If there is only one EQ p^* observed, we cannot identify β_1 and α separately.
  - If two EQ are observed, exactly identified.
  - If more than two EQ are observed, overidentified.
- Intuition?
  - Rule out parameters that do not support all the EQ

Figure 3: Identification



- Assume only one EQ p^* is played.
  - Prob that n players take action 1 out of N players is

$$\mathrm{Pr}(n) = \frac{N!}{n!(N-n)!}(p^*)^n(1-p^*)^{N-n}$$

  - If the assumption is correct, estimate p^* by

$$p^* = \text{average number of players taking action 1}$$

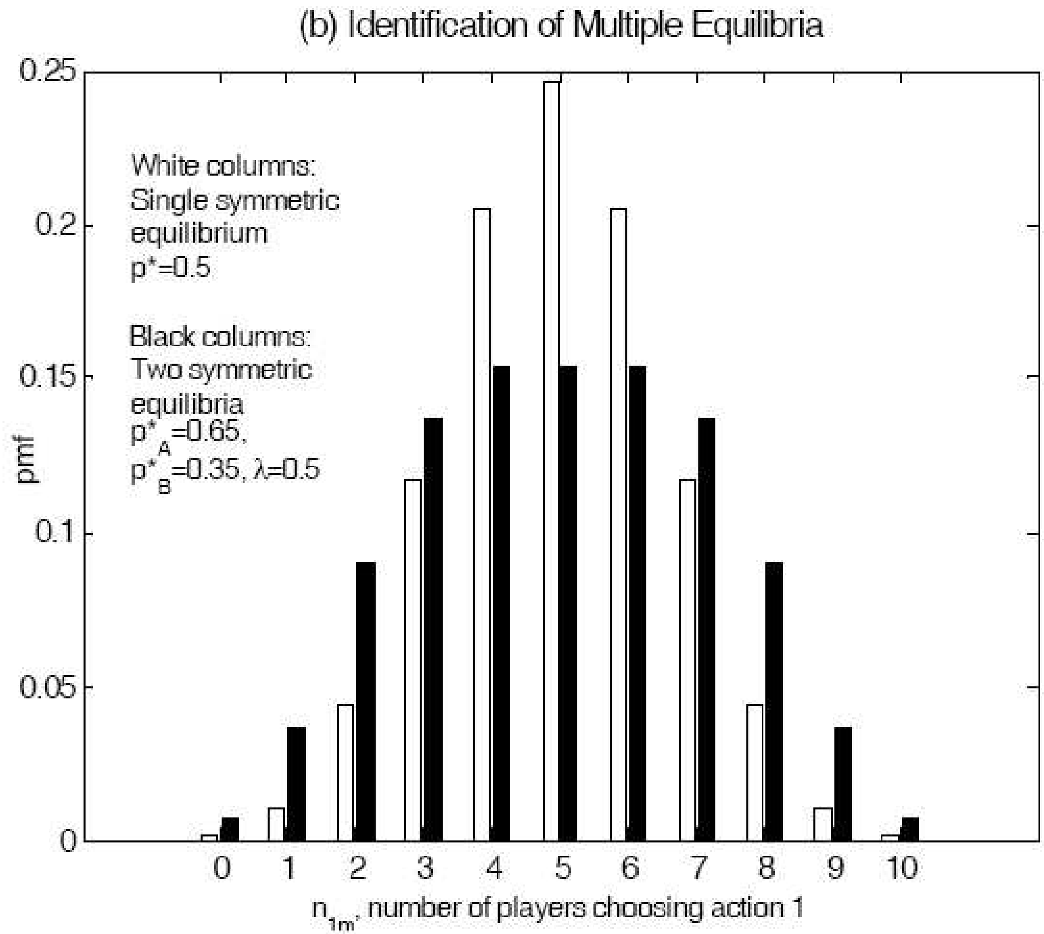
- Assume two EQ p_1^* and p_2^* are played.
  - λ_1 portion of players choose p_1^*, and λ_2 choose p_2^*

$$\begin{aligned}\Pr(n) &= \lambda_1 \frac{N!}{n!(N-n)!}(p_1^*)^n(1-p_1^*)^{N-n} \\ &+ \lambda_2 \frac{N!}{n!(N-n)!}(p_2^*)^n(1-p_2^*)^{N-n}\end{aligned}$$

- Generally assume E equilibria are played.

$$\Pr(n) = \sum_{e=1}^{E} \lambda_e \frac{N!}{n!(N-n)!}(p_e^*)^n(1-p_e^*)^{N-n}$$

Figure 3: Identification



bigger variance with 2 EQ than with 1 EQ

- Identification of EQ p^*
  - If we have markets with more than 3 players,
    we can identify 2 EQ (so can identify β_1 and α).
  - How?
    Since we have probability equation, use MLE.

$$\max_{\lambda_1, p_1^*, p_2^*} \quad \log L \equiv \sum_{m=1}^{M} \log \Pr(n_m)$$

  - Then, use $\widehat{p_1^*}$ and $\widehat{p_2^*}$ to get β_1 and α.

$$p^* = \frac{e^{\beta_1 + \alpha p^*}}{e^{\beta_1 + \alpha p^*} + e^{\alpha(1-p^*)}}$$

- But, how do we know that there are 2 EQ?
  - Follow a method proposed by Chen et al. (2001)

$$\max_{\lambda_1, p_1^*, p_2^*} \quad l^m \equiv \log L + C \log(4\lambda[1-\lambda])$$

(1) Choose appropriate C

(2) Find unrestricted λ_1, p_1^* and p_2^*

(3) Find restricted p^* under $H_0 : \lambda = \frac{1}{2}$ and $p_1^* = p_2^*$.

- Perform likelihood ratio test using that under H_0,

$$2\left[l^m_{\text{unrestricted}} - l^m_{\text{restricted}}\right] \dot{\sim} \frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$$

- Empirical result using radio commercial markets
  - Commercials aired at the same time. Why?
    1. Network externalities, i.e., $\alpha > 0$.
    2. Want to avoid quarter-hour for commercials.
  - n is dispersed widely. Why?
    1. Multiple equilibria
    2. Heterogeneity in β_1

- Data description
  - 1,094 contemporary music stations
  - 147 different markets
  - first five weekdays of each month in 2001

- Analysis
  - whether commercials aired during :48-:52 or :53-:57
  - drivetime 4-5 pm, 5-6 pm
  - non-drivetime 12-1 pm, 9-10 pm

- Logit estimation
  - Action 1 = commercial aired during :53-:57

$$\Pr(\text{Action}_i = 1) = \frac{e^{\alpha P_{-i1} + X\beta}}{1 + e^{\alpha P_{-i1} + X\beta}}$$

  - Problematic. Why?
    Systematically yields $\alpha > 0$.
  - See Table 4.

- Test of multiple equilibria
  - 2 equilibria during drivetime against 1 equilibrium
  - 1 equilibrium during non-drivetime against 2 equilibria
  - Different test
    - Using only large markets, 1 equilibrium in all cases
    - Using only small markets, 2 or 4 equilibria during drive-time
  - See Table 5.

Table 6: Results from the Basic Model with the Parameters Constant Across Market-Days

	Drivetime Hours		Non-Drivetime Hours	
	4-5 pm	5-6 pm	12-1 pm	9-10 pm
(a) One equilibrium model				
β_1 (assuming $\alpha = 0$)	-0.0874 (0.0483)	-0.0546 (0.0491)	-0.1040 (0.0387)	0.0771 (0.0344)
Log-likelihood	-20995.6	-21539.0	-20181.1	-19435.8
Implied equilibrium p^*	0.4782	0.4863	0.4740	0.5193
(b) Two equilibria model with non-stable equilibria allowed				
β_1	-0.0008 (0.0050)	0.0043 (0.1096)	-	-0.4138 (0.2287)
α	2.0151 (0.1045)	2.0796 (2.6434)	-	14.0160 (3.8203)
λ	0.5269 (0.2627)	0.0657 (0.3432)	-	0.0046 (0.3387)
Log-likelihood	-20989.6	-21535.0	-	-19434.4
Implied equilibria p_A^*, p_B^*	(0.535,0.414)	(0.678,0.473)	-	(1,0.5172)
(c) Two equilibria model with only stable equilibria allowed				
β_1	-0.0007 (0.0011)	0.0008 (0.0009)	-	0.0001 (0.0005)
α	2.0130 (0.0059)	2.0141 (0.0071)	-	2.0023 (0.0052)
λ	0.4809 (0.1161)	0.2214 (0.1071)	-	0.7147 (0.1855)
Log-likelihood	-20989.6	-21535.2	-	-19435.7
Implied equilibria p_A^*, p_B^*	(0.540,0.419)	(0.583,0.458)	-	(0.534,0.483)
Joint payoff maximizing p^{JP}	0.021	0.979	-	0.979
Number of market-days	7,656	7,702	7,549	7,482
Number of station-days	30,332	31,091	29,172	28,070

- Test of heterogeneity in β_1

  – Not likely. Why?

  (1) Only 2 EQ rather than many EQ

  (2) Include some observed characteristics and allow heterogeneity in β_1. Then,

  * Log-likelihood does not change significantly.

  * Variance in β_1 is too small.

  – See Table 8.

- Concluding remarks
  - Multiple equilibria help with identification.
  - Rule out parameters that do not support all the EQ.
  - In radio commercial markets, multiple EQ exist.
    It's not likely to be heterogeneity in β_1.
  - Estimation result: network effect exists ($\alpha > 0$).