ECON 271B assignment #1

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Differentiated Products Analysis: Analytic Models

(1) OLS with LOGIT model

With the utility specification as

$$U_{ij} = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \xi_j + \varepsilon_{ij}$$

the share of product j is given by

$$s_j = \frac{e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j}}{1+\sum_c e^{\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c}}$$

and the share of outside option is given by

$$s_0 = \frac{1}{1+\sum_c e^{\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c}}$$

Dividing side by side, we get

$$\frac{s_j}{s_0} = e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j}$$

Taking log yields Berry inversion of LOGIT model as

$$\log\frac{s_j}{s_0} = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \xi_j$$

Using $\log\frac{s_j}{s_0}$ as a dependent variable, and constant, x_j, g_j and p_j as explanatory variables, run OLS under the assumption that $E[\xi_j|x_j, g_j, p_j] = 0$, then we have

coefficient	β_0	β_1	β_2	α
estimate	-3.0383	0.7783	0.0621	0.7110
(s.e.)	(0.4208)	(0.0265)	(0.0446)	(0.2150)

Clearly, OLS does not provide reasonable estimates. In theory, α would be negative, but OLS estimate of α is positive.

(2) Unreasonable estimates of OLS result from the assumption that $E[\xi_j|p_j] = 0$. Actually, when p_j is high, it is likely that unobservable characteristic of the commodity is also high, which means that ξ_j and p_j are positively correlated. If we assume uncorrelatedness between them, α, a coefficient of p_j would be biased upward. We have to use instrumental variables to overcome endogeneity problem. Two candidates are a marginal cost shifter, w_j, and the total number of products in the same group, z_j. For these to be vaild instruments, they should be correlated much with p_j, but uncorrelated with ξ_j.

First, they seem to be correlated much with p_j. High w_j is likely to increase cost, and thus price accordingly. Also, if there are more products in the same group, it expedites competition among them, which decreases price. Second, there would be some debates about uncorrelatedness between ξ_j and the two candidates. In many cases, they do not seem to be correlated with ξ_j, since input price and competition are not likely to affect ξ_j. But in some sense, they may influence ξ_j. If we interpret ξ_j as quality of the product, invention of a new technology clearly reduces cost and is quite likely to raise the quality. Also, increased competition among firms may result in increase in the quality as well as increase in observable attributes of the product. So uncorrelatedness between ξ_j and the two candidates are not clear.

(3) IV with LOGIT model

We use w_j as an instrument of p_j to get the following result.

coefficient	β_0	β_1	β_2	α
estimate (s.e.)	-0.2587 (0.8079)	0.7985 (0.0297)	0.0656 (0.0493)	-0.7135 (0.4136)

There is no big difference in β_1 and β_2 between the OLS and the IV results, but β_0 and α has changed much. Notably, α is negative here, which seems reasonable. To calculate own and cross price elasticities, differentiate s_j with respect to p_j, then

$$\frac{ds_j}{dp_j} = \alpha \frac{e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j}}{1+\sum_c e^{\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c}} - \alpha \left(\frac{e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j}}{1+\sum_c e^{\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c}} \right)^2 = \alpha s_j(1-s_j)$$

and differentiate s_k with respect to p_j, then

$$\frac{ds_k}{dp_j} = -\alpha \frac{e^{\beta_0+\beta_1 x_k+\beta_2 g_k+\alpha p_k+\xi_k} e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j}}{(1+\sum_c e^{\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c})^2} = -\alpha s_k s_j$$

So price elasticities are obtained as

$$\begin{aligned} e_{s_j,p_j} &= \frac{ds_j}{dp_j} \cdot \frac{p_j}{s_j} = \alpha p_j (1-s_j) \\ e_{s_k,p_j} &= \frac{ds_k}{dp_j} \cdot \frac{p_j}{s_k} = -\alpha p_j s_j \end{aligned}$$

From these, we can calculate the implied price-cost markups. It can be derived from FOC of profit maximizing problem. Each firm maximizes its profit

$$\max_{p_j} \ (p_j - mc_j)s_j$$

so FOC reads as

$$s_j + (p_j - mc_j)\frac{ds_j}{dp_j} = 0$$

Rearranging, we have

$$p_j - mc_j = -s_j \frac{dp_j}{ds_j} = -\frac{p_j}{e_{s_j,p_j}} = -\frac{1}{\alpha(1-s_j)}$$

Obtained elasticities and markups are as follows.

Own and Cross Price Elasticities

		change in shares			
		s_{77}	s_{78}	s_{79}	s_{80}
change in price	p_{77}	-1.3907	0.0875	0.0875	0.0875
	p_{78}	0.0496	-1.2617	0.0496	0.0496
	p_{79}	0.0644	0.0644	-1.3898	0.0644
	p_{80}	0.0464	0.0464	0.0464	-1.2670
Group		0	1	0	1
Markups		1.4896	1.4565	1.4664	1.4528

Within and across group, one commodity has the same cross price elasticities for all the other commodities, and the markups are almost the same for all commodities. These results are not appealing. Elasticities and markups depend only on their own price and share as well as α. But they do not depend on group characteristic, nor on market characteristics.

(4) IV with nested LOGIT model

With the utility specification as

$$U_{ij} = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \xi_j + \rho \zeta_{ig_j} + (1-\rho)\varepsilon_{ij}$$

the share of product j is given by

$$s_j = s_{j|g} s_g = \frac{e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j)}}{\sum_{c\in g_j} e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c)}} \cdot \frac{\left[\sum_{c\in g_j} e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c)}\right]^{1-\rho}}{1+\sum_{g=0}^{1}\left[\sum_{c\in g} e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c)}\right]^{1-\rho}}$$

and the share of outside option is given by

$$s_0 = \frac{1}{1+\sum_{g=0}^{1}\left[\sum_{c\in g} e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c)}\right]^{1-\rho}}$$

Dividing side by side,

$$\frac{s_j}{s_0} = \frac{e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j)}}{\left[\sum_{c\in g_j} e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c)}\right]^{\rho}}$$

or

$$\frac{s_j}{s_0} = e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j}\left[\frac{e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\xi_j)}}{\sum_{c\in g_j} e^{\frac{1}{1-\rho}(\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\xi_c)}}\right]^{\rho}$$

Taking log yields Berry inversion of nested LOGIT model as

$$\log\frac{s_j}{s_0} = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \rho \log s_{j|g} + \xi_j$$

This is an econometric model which explains $\log\frac{s_j}{s_0}$ with constant, x_j, g_j, p_j and $\log s_{j|g}$. There is an endogeneity problem since p_j is correlated with ξ_j, and also $\log s_{j|g}$ seems to be correlated with ξ_j. High within-group share of a product is likely to reflect the fact that the product has high quality. So we need another instrument to be used for $\log s_{j|g}$. Use z_j as a new instrument and run IV estimation under the assumption that $E[\xi_j|x_j, g_j, w_j, z_j] = 0$, then we have

coefficient	β_0	β_1	β_2	α	ρ
estimate	0.4597	0.6471	0.0805	-0.5095	0.4630
(s.e.)	(0.5998)	(0.0235)	(0.0347)	(0.2824)	(0.0395)

The estimates of all the coefficients β_0, β_1, β_2 and α differ a little from those in LOGIT model. This is because a group specific fixed effect explains some variation of the share. It is interesting that α becomes less when we use nested LOGIT model, which means that if this model is true, the effect of price on share was overestimated in the LOGIT model. Actually, ρ is statistically highly significant, so it is likely that there is a group effect.
We may use other instrumental variables to obtain less variance of estimates. A potential instrument suggested by Nevo (2001) is price of the same commodity sold in the other markets. It is clearly correlated with p_j, but it is not correalted with ξ_j, if we think of ξ_j as a market specific unobservable characteristic of the commodity. But this assumption fails if ξ_j reflects some attributes which do not change across the markets.

(5) For convenience, define first $\delta_j = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \xi_j$, and $A_g = \sum_{c \in g} e^{\frac{\delta_c}{1-\rho}}$.

$$\begin{aligned}
\log s_j &= \frac{\delta_j}{1-\rho} - \log A_{g_j} + (1-\rho)\log A_{g_j} - \log\left(1 + \sum_{g=0}^{1} A_g^{1-\rho}\right) \\
&= \frac{\delta_j}{1-\rho} - \rho \log A_{g_j} - \log\left(1 + \sum_{g=0}^{1} A_g^{1-\rho}\right)
\end{aligned}$$

Differentiate $\log s_j$ with respect to p_j, then

$$\begin{aligned}
\frac{d\log s_j}{d\delta_j} &= \frac{1}{1-\rho} - \rho \cdot \frac{\frac{1}{1-\rho} e^{\frac{\delta_j}{1-\rho}}}{A_{g_j}} - \frac{(1-\rho)A_{g_j}^{-\rho} \cdot \frac{1}{1-\rho} e^{\frac{\delta_j}{1-\rho}}}{1 + \sum_{g=0}^{1} A_g^{1-\rho}} \\
&= \frac{1}{1-\rho} - \frac{\rho}{1-\rho} \cdot \frac{e^{\frac{\delta_j}{1-\rho}}}{A_{g_j}} - \frac{e^{\frac{\delta_j}{1-\rho}}}{A_{g_j}} \cdot \frac{A_{g_j}^{1-\rho}}{1 + \sum_{g=0}^{1} A_g^{1-\rho}} \\
&= \frac{1}{1-\rho} - \frac{\rho}{1-\rho} s_{j|g} - s_j
\end{aligned}$$

Now let k be the product in the same group with j. Then the first term in $\log s_k$ disappears when differentiating with respect to p_j. Note that $A_{g_k} = A_{g_j}$, so

$$\frac{d\log s_k}{d\delta_j} = -\frac{\rho}{1-\rho} s_{j|g} - s_j$$

Let k be the product in the different group from j. Then the first two terms in $\log s_k$ disappear when differentiating, so

$$\frac{d\log s_k}{d\delta_j} = -s_j$$

Therefore, the own and cross price elasticities are

$$\begin{aligned} e_{s_k,p_j} &= \frac{ds_k}{dp_j}\cdot\frac{p_j}{s_k} = \frac{p_j}{s_k}\cdot\frac{ds_k}{d\delta_j}\cdot\frac{d\delta_j}{dp_j} = p_j\frac{d\log s_k}{d\delta_j}\frac{d\delta_j}{dp_j} = \alpha p_j\frac{d\log s_k}{d\delta_j} \\ &= \begin{cases} \alpha p_j\left(\frac{1}{1-\rho} - \frac{\rho}{1-\rho}s_{j|g} - s_j\right) & \text{when } k=j \\ \alpha p_j\left(-\frac{\rho}{1-\rho}s_{j|g} - s_j\right) & \text{when } k \text{ is in the same group with } j \\ -\alpha p_j s_j & \text{when } k \text{ and } j \text{ are not in the same group} \end{cases} \end{aligned}$$

The implied price-cost markups are again

$$p_j - mc_j = -\frac{p_j}{e_{s_j,p_j}} = -\frac{1}{\alpha\left(\frac{1}{1-\rho} - \frac{\rho}{1-\rho}s_{j|g} - s_j\right)}$$

Obtained elasticities and markups are as follows.

Own and Cross Price Elasticities

		change in shares			
		s_{77}	s_{78}	s_{79}	s_{80}
change in price	p_{77}	-1.5604	0.0625	**0.4050**	0.0625
	p_{78}	0.0354	-1.6608	0.0354	**0.0828**
	p_{79}	**0.2980**	0.0460	-1.6355	0.0460
	p_{80}	0.0331	**0.0774**	0.0331	-1.6689
Group		0	1	0	1
Markups		1.3276	1.1065	1.2461	1.1029

We can see that change in price of a product affects the share of the product in the same group more than that in different group. Bold numbers are greater than the others in the same row. Own price elasticities are greater in absolute values than before. Markups differ for each product, and especially, products in group 1 have less markups than those in group 0. Investigating market 4, which contains products 61 to 80, there are 17 group 1 products and only 3 group 0 ones. So it is likely that there is more competition among group 1 products than among group 0 ones. This is consistent with the result that markups are less for products in group 1.

A Simple Random Coefficients Model

(6) In this model, consumer's utility can be written as

$$U_{ij} = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \sigma\widetilde{\beta}_{2i} g_j + \varepsilon_{ij}$$

In this model, there is extra randomness in utility for products in group 1, which is denoted by $\sigma\widetilde{\beta}_{2i}g_j$. In contrast, in the nested LOGIT model, a random term $\rho\zeta_{ig}$ affects both products in group 1 and those in group 0. This, however, is a difference between these two specific models. In general, random coefficients model have much more freedom in specifying groupwise randomness. For example, we can add another term to write

$$U_{ij} = \beta_0 + \beta_1 x_j + \beta_2 g_j + \alpha p_j + \sigma\widetilde{\beta}_{2i}g_j + \phi\widetilde{\beta}_{3i}(1-g_j) + \varepsilon_{ij}$$

where $\widetilde{\beta}_{3i}$ is a random variable. This specification is a generalized version of the nested LOGIT model. Imposing two restrictions that $\sigma = \phi = \rho$ and that $\widetilde{\beta}_{2i}$ and $\widetilde{\beta}_{3i}$ have the same distribution with ζ_{ig} yields the nested LOGIT model.

(7) By integrating out with respect to ε_{ij}, we can write

$$\begin{aligned} s_j &= \int\int I(U_{ij} \geq U_{ic}\ \forall c)p(d\varepsilon_{ij})p(d\widetilde{\beta}_{2i}) + \xi_j \\ &= \int \frac{e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\sigma\widetilde{\beta}_{2i}g_j}}{1+\sum_c e^{\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\sigma\widetilde{\beta}_{2i}g_c}} p(d\widetilde{\beta}_{2i}) + \xi_j \end{aligned}$$

Simulating integral with random draws of $\widetilde{\beta}_{2i}$ from $N(0,1)$ gives us simulated $\widehat{\xi}_j$.

$$\widehat{\xi}_j = s_j - \frac{1}{N_S}\sum_{i=1}^{N_s} \frac{e^{\beta_0+\beta_1 x_j+\beta_2 g_j+\alpha p_j+\sigma\widetilde{\beta}_{2i}g_j}}{1+\sum_c e^{\beta_0+\beta_1 x_c+\beta_2 g_c+\alpha p_c+\sigma\widetilde{\beta}_{2i}g_c}}$$

where N_s is the number of simulations. Using the conditional moment condition $E[\xi_j|x_j, g_j, w_j, z_j] = 0$,

$$G(\theta) = E\left[\xi_j \begin{pmatrix} 1 \\ x_j \\ g_j \\ w_j \\ z_j \end{pmatrix}\right] = 0$$

Based on this, define its sample analogue by

$$G_N(\theta) = \frac{1}{N}\sum_j h_j(\theta) \equiv \frac{1}{N}\sum_j \widehat{\xi}_j \begin{pmatrix} 1 \\ x_j \\ g_j \\ w_j \\ z_j \end{pmatrix}$$

and use this to run GMM estimation by minimizing

$$G_N(\theta)' A G_N(\theta)$$

where we usually use

$$A = Var\Big(G_N(\theta)\Big)^{-1}$$

to minimize variance of GMM estimate $\widehat{\theta}_{GMM}$. Note

$$Var\Big(G_N(\theta)\Big) = \frac{1}{N^2}\sum_j Var\Big(h_j(\theta)\Big) = \frac{1}{N}Var\Big(h_j(\theta)\Big) = \frac{1}{N}E[h_j(\theta)h_j(\theta)']$$

since data are iid and $E[h_j(\theta)] = 0$. We replace an expectatioin by its sample analogue

$$\widehat{V} = \frac{1}{N^2}\sum_j h_j(\theta)h_j(\theta)'$$

In principal, choice of weighting matrix A does not affect estimates of θ, since there are 5 unknowns and 5 moment conditions. But due to complex form of the objective function and inaccuracy of numerical minimization routine, estimates of θ depend on choice of A and the initial guess θ_0. First LOGIT IV result and $\sigma = 0$ are used as θ to get

$$A(\theta) = \left(\frac{1}{N^2}\sum_j h_j(\theta)h_j(\theta)'\right)^{-1}$$

and $\theta_0 = 0$ is used as an initial guess for numerical minimization routine. After obtaining $\widehat{\theta}$, recalcualte A and repeat minimization routine by using $\widehat{\theta}$ as an intinal guess. Iteration continues until $\widehat{\theta}$ does not change much between iterations. The result is given in the following table.

coefficient	β_0	β_1	β_2	α	σ
1st iteration	-2.710912	0.836676	0.198267	0.495596	1.435237
2nd iteration	-0.004136	0.982125	0.384453	-0.790314	2.954734
3rd iteration	-0.004175	0.982118	0.384447	-0.790293	2.954688
4th iteration	-0.004149	0.982120	0.384450	-0.790307	2.954688
5th iteration	-0.004149	0.982120	0.384450	-0.790307	2.954688

In the following table, given are the results from various choices of A and θ_0. The first line is provided for comparison. In some cases, we obtain undesirable results.

initial θ for A		initial guess θ_0		#	estimates of coefficients				
$\beta_0, \beta_1, \beta_2, \alpha$	σ	$\beta_0, \beta_1, \beta_2, \alpha$	σ	iter	β_0	β_1	β_2	α	σ
LOGIT IV	0	0	0	5	-0.0041	0.9821	0.3845	-0.7903	2.9547
LOGIT IV	0	LOGIT IV	0	5	-0.0029	0.9817	-0.4046	-0.7910	-2.9644
LOGIT IV	1	0	1	4	-0.0042	0.9821	0.3844	-0.7903	2.9547
LOGIT IV	1	LOGIT IV	1	4	-0.0041	0.9821	0.3844	-0.7903	2.9547
nested IV	0	0	0	5	-0.0029	0.9817	-0.4046	-0.7910	-2.9644
nested IV	0	nested IV	0	2	-0.0041	0.9821	0.3844	-0.7903	2.9547
nested IV	1	0	1	4	-0.0042	0.9821	0.3845	-0.7903	2.9547
nested IV	1	nested IV	1	4	-0.0042	0.9821	0.3845	-0.7903	2.9547
$A = I_5$		nested IV	0	5	-0.0042	0.9821	0.3844	-0.7903	2.9547
$A = I_5$		nested IV	0.463	4	-0.0029	0.9817	-0.4046	-0.7910	-2.9644
$A = I_5$		nested IV	1	4	-0.0041	0.9821	0.3845	-0.7903	2.9547

(8) Letting $Q_N(\theta) = \frac{1}{2}G_N(\theta)'AG_N(\theta)$, GMM estimate $\widehat{\theta}$ satisfies

$$0 = \frac{\partial Q_N(\widehat{\theta})}{\partial \theta} = \frac{\partial Q_N(\theta)}{\partial \theta} + \frac{\partial^2 Q_N(\widetilde{\theta})}{\partial \theta \partial \theta'}(\widehat{\theta} - \theta)$$

where $\widetilde{\theta}$ is between $\widehat{\theta}$ and θ. So

$$\sqrt{N}(\widehat{\theta} - \theta) = -\left(\frac{\partial^2 Q_N(\widetilde{\theta})}{\partial \theta \partial \theta'}\right)^{-1} \cdot \sqrt{N}\frac{\partial Q_N(\theta)}{\partial \theta}$$

Assume that there is no randomness from simulation of shares and define

$$\begin{aligned} \Gamma &= E\left[\frac{\partial G_N(\theta)}{\partial \theta'}\right] = E\left[\frac{\partial h_j(\theta)}{\partial \theta'}\right] \\ \Omega &= E\left[NG_N(\theta)G_N(\theta)'\right] = Var\left(\sqrt{N}G_N(\theta)\right) = E\left[h_j(\theta)h_j(\theta)'\right] \end{aligned}$$

then, we have

$$\begin{aligned} \frac{\partial^2 Q_N(\widetilde{\theta})}{\partial \theta \partial \theta'} &\xrightarrow{p} \Gamma' A \Gamma \\ \sqrt{N}\frac{\partial Q_N(\theta)}{\partial \theta} &\xrightarrow{d} N\Big(0, \Gamma' A \Omega A \Gamma\Big) \end{aligned}$$

So we can approximate the variance of $\widehat{\theta}$ as

$$Var(\widehat{\theta}) = \left(\widehat{\Gamma}' A \widehat{\Gamma}\right)^{-1} \widehat{\Gamma}' A \frac{1}{N} \widehat{\Omega} A \widehat{\Gamma} \left(\widehat{\Gamma}' A \widehat{\Gamma}\right)^{-1}$$

where

$$\begin{aligned} \widehat{\Gamma} &= \frac{1}{N}\sum_j \frac{\partial h_j(\widehat{\theta})}{\partial \theta'} \\ \frac{1}{N}\widehat{\Omega} &= \frac{1}{N^2}\sum_j h_j(\widehat{\theta})h_j(\widehat{\theta})' \end{aligned}$$

Using this, standard errors are obtained as follows.

coefficient	β_0	β_1	β_2	α	σ
estimate	-0.0041	0.9821	0.3845	-0.7903	2.9547
(s.e.)	(0.1252)	(0.0114)	(0.0155)	(0.0627)	(0.0684)

(9) Considering randomness generated by simulation,

$$Var(G_N(\theta)) = E\left[G_N(\theta)G_N(\theta)'|\overline{P_S}\right] + E\left[G_N(\theta)G_N(\theta)'|\overline{W}\right]$$

Approximate expectations by their sample analogues, then

$$\frac{1}{N}\widehat{\Omega} = \frac{1}{N^2}\sum_j h_j(\widehat{\theta})h_j(\widehat{\theta})' + \frac{1}{B}\sum_S G_N(P_S, \widehat{\theta})G_N(P_S, \widehat{\theta})'$$

where B is the number of bootstrap samples for simulation. Draw 100 sets of random numbers P_S from $N(0, I_{50})$, use these to get the second term, and then compute

$$Var(\widehat{\theta}) = \left(\widehat{\Gamma}' A \widehat{\Gamma}\right)^{-1} \widehat{\Gamma}' A \frac{1}{N} \widehat{\Omega} A \widehat{\Gamma} \left(\widehat{\Gamma}' A \widehat{\Gamma}\right)^{-1}$$

Using this, standards errors are as follows.

coefficient	β_0	β_1	β_2	α	σ
estimate	-0.0041	0.9821	0.3845	-0.7903	2.9547
(s.e.)	(0.1253)	(0.0117)	(0.6244)	(0.0628)	(0.7084)

Standard errors for β_0, β_1, and α do not change much, but those for β_2 and σ change dramatically. This is interesting. Note that the only random coefficient is β_{2i}, and we characterize it into two parameters β_2 and σ. If we could calculate shares analytically without simulation, standard errors would be ones obtained in (8). Simulation generates randomness, which makes β_2 and σ difficult to be identified from each other. If this is correct, we can increase the number of simulations to get more variations in $\widetilde{\beta}_{2i}$ and to identify σ from β_2. The law of large numbers would ensure that standard errors of β_2 and σ decrease as NS increases.

The following two additional experiments support the above explanation. Incrseaing B does not affect the result, so simulation errors are captured properly from bootstrapping. But when we increase NS, standard errors decrease significantly. Obviously, this is because using more simulation draws averages simulation errors out. For example, with $NS = 500$, standard errors are as follows.

coefficient	β_0	β_1	β_2	α	σ
estimate	-0.0064	0.9803	-0.0156	-0.7897	2.4746
s.e. ignoring simulation error	(0.1357)	(0.0124)	(0.0146)	(0.0680)	(0.0598)
s.e. including simulation error	(0.1358)	(0.0126)	(0.1201)	(0.0680)	(0.1294)