

ECON 271C homework #2
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RED texts are comments by the professor.

BLUE texts are my correction and friends' advices.

Industry equilibrium

7. Industry equilibrium 9/10 points (a)

I assume that a private firm becomes public as soon as it pays c_0 . Denote private firm's current period profit and present value by π^{PR} and V^{PR} , respectively. π and V without any superscript stand for public firm's profit function and value function, respectively. Private firms have a capacity constraint, while public firms do not, so

$$\pi^{\text{PR}}(\varphi, p) \leq \pi(\varphi, p) \quad \text{for any } \varphi \text{ and } p$$

with equality when a capacity constraint is slack. Then, value functions would satisfy

$$V(\varphi, p) = \max \left\{ 0, \pi(\varphi, p) + \delta \int V(\varphi', p) F(d\varphi' | \varphi) \right\}$$

$$V^{\text{PR}}(\varphi, p) = \max \left\{ 0, \pi^{\text{PR}}(\varphi, p) + \delta \int V^{\text{PR}}(\varphi', p) F(d\varphi' | \varphi), \pi(\varphi, p) - c_0 + \delta \int V(\varphi', p) F(d\varphi' | \varphi) \right\}$$

Also, there is a value function for an entrant. I assume that a firm behaves as a private firm the period when it enters.

$$V^e(p) = \int V^{\text{PR}}(\varphi, p) G(d\varphi) - c_e$$

For an outside firm to be indifferent between entering and staying out, it should be that $V^e(p^*) = 0$, or equivalently,

$$\int V^{\text{PR}}(\varphi, p^*) G(d\varphi) = c_e \tag{1}$$

Suppose that there is no ex post uncertainty, in the sense that an existing firm has the same φ forever. Then, the value functions would be

$$V(\varphi, p) = \max \left\{ 0, \frac{\pi(\varphi, p)}{1 - \delta} \right\}$$

$$V^{\text{PR}}(\varphi, p) = \max \left\{ 0, \frac{\pi^{\text{PR}}(\varphi, p)}{1 - \delta}, \frac{\pi(\varphi, p)}{1 - \delta} - c_0 \right\}$$

In this case, there exist two thresholds φ^* and φ^{**} such that a private firm who gets φ decides to

exit	if $\varphi < \varphi^*$,
stay as private	if $\varphi^* \leq \varphi < \varphi^{**}$, and
pay c_0 and become public	if $\varphi \geq \varphi^{**}$

These thresholds satisfy

$$\begin{aligned}\frac{\pi^{\text{PR}}(\varphi^*, p^*)}{1 - \delta} &= 0 \\ \frac{\pi^{\text{PR}}(\varphi^{**}, p^*)}{1 - \delta} &= \frac{\pi(\varphi^{**}, p^*)}{1 - \delta} - c_0\end{aligned}$$

A stationary equilibrium would be $(p^*, \varphi^*, \varphi^{**}, M^*)$ that satisfies (1), the above two equations, and the following condition of an invariant measure of firms.

$$D \left(\int_{\varphi^*}^{\varphi^{**}} x^{\text{PR}}(\varphi, p^*) M^* G(d\varphi) + \int_{\varphi^{**}} x(\varphi, p^*) M^* G(d\varphi) \right) = p^*$$

Now consider the case where φ changes over time. In this case, a public firm can exit if it gets a very bad shock. So there are three thresholds φ^* , φ^{**} and φ^{***} . The first two are defined as before and φ^{***} is such that a public firm exits when it gets $\varphi < \varphi^{***}$. These lead to the following equations.

$$\pi^{\text{PR}}(\varphi^*, p^*) + \delta \int V^{\text{PR}}(\varphi', p^*) F(d\varphi' | \varphi^*) = 0 \quad (2)$$

$$\pi^{\text{PR}}(\varphi^{**}, p^*) + \delta \int V^{\text{PR}}(\varphi', p^*) F(d\varphi' | \varphi^{**}) = \pi(\varphi^{**}, p^*) - c_0 + \delta \int V(\varphi', p^*) F(d\varphi' | \varphi^{**}) \quad (3)$$

$$\pi(\varphi^{***}, p^*) + \delta \int V(\varphi', p^*) F(d\varphi' | \varphi^{***}) = 0 \quad (4)$$

From these conditions, we can see that $\varphi^{***} \leq \varphi^* \leq \varphi^{**}$. Also there are two measures, one for private firms and the other for public firms. Denote this as μ^{PR} and μ , respectively. In a stationary equilibrium, these should be invariant, and thus satisfy

$$\mu^{\text{PR}}([\varphi^*, s]) = \lambda^* [G(s) - G(\varphi^*)] + \int [F(s|\varphi) - F(\varphi^*|\varphi)] \mu^{\text{PR}}(d\varphi) \quad \text{for } s \leq \varphi^{**} \quad (5)$$

$$\begin{aligned}\mu([\varphi^{***}, s]) &= \lambda^* \max \{0, G(s) - G(\varphi^{**})\} + \int \max \{0, F(s|\varphi) - F(\varphi^{**}|\varphi)\} \mu^{\text{PR}}(d\varphi) \\ &\quad + \int [F(s|\varphi) - F(\varphi^{***}|\varphi)] \mu(d\varphi)\end{aligned} \quad (6)$$

Under the above measures, p^* should support total demand.

$$D \left(\int_{\varphi^*}^{\varphi^{**}} x^{\text{PR}}(\varphi, p^*) \mu^{\text{PR}}(d\varphi) + \int_{\varphi^{***}} x(\varphi, p^*) \mu(d\varphi) \right) = p^* \quad (7)$$

A stationary equilibrium is $(V^{\text{PR}}, V, p^*, \varphi^*, \varphi^{**}, \varphi^{***}, \mu^{\text{PR}}, \mu, \lambda^*)$ that satisfies equations (1) to (7) as well as the definition of value functions.

(b)

It is clear that value functions V^{PR} and V are uniquely defined under some assumptions on π^{PR} , π and F . Also φ^* , φ^{**} , φ^{***} and p^* are uniquely defined by equations (1) to (4) given V^{PR} and V . Equations (5) to (7) characterize unique λ^* , and thus μ^{PR} and μ as well. **(How do we know?)**

Therefore, there exists a unique stationary equilibrium if μ^{PR} and μ are uniquely defined by λ^* . Temporarily suppose that the support of measures are discrete, and transform (5) and (6) into equations in vector version of measures.

$$\begin{aligned}\mu^{\text{PR}} &= \lambda^* \nu^{\text{PR}} + T_1 \mu^{\text{PR}} \\ \mu &= \lambda^* \nu + S \mu^{\text{PR}} + T_2 \mu\end{aligned}$$

This leads to the following solution.

$$\begin{aligned}\mu^{\text{PR}} &= (I - T_1)^{-1} \lambda^* \nu^{\text{PR}} = \lambda^* \sum_{n=0}^{\infty} T_1^n \nu^{\text{PR}} \\ \mu &= (I - T_2)^{-1} (\lambda^* \nu + S \mu^{\text{PR}}) = \sum_{n=0}^{\infty} T_2^n (\lambda^* \nu + S \mu^{\text{PR}})\end{aligned}$$

Analogously the measure functions would satisfy

$$\begin{aligned}\mu^{\text{PR}} &= \lambda^* \sum_{t=0}^{\infty} \alpha_{1t} \tilde{\mu}_{1t} \\ \mu &= \lambda^* \sum_{t=0}^{\infty} \alpha_{2t} \tilde{\mu}_{2t} + \sum_{t=0}^{\infty} \alpha_{3t} \tilde{\mu}_{3t}\end{aligned}$$

where $\tilde{\mu}_{1t}$ is a conditional cdf of private firms' shock after t periods of survival, $\tilde{\mu}_{2t}$ is a conditional cdf of public firms' shock after t periods of survival, $\tilde{\mu}_{3t}$ is a conditional cdf of shock of private firms becoming public after t periods of survival, and α_{1t}, α_{2t} and α_{3t} are corresponding survival probabilities. These measure functions exist and are unique if

$$\sum_{t=0}^{\infty} \alpha_{1t} < \infty, \quad \sum_{t=0}^{\infty} \alpha_{2t} < \infty \quad \text{and} \quad \sum_{t=0}^{\infty} \alpha_{3t} < \infty$$

(c)

As shown above, φ^{***} is smaller than φ^* . So the interval in which private firms exist is narrower than and is included in the interval where public firms exist. Also the average of the profit made by public firms is higher than that made by private firms. (How can you be sure?) This is because public firms have a positive probability that they get a very high shock in the future. In the stationary equilibrium, it should be the case that the change rate of private firms into public firms is the same with the exit rate of public firms, and also that the change rate plus the exit rate of private firms is the same with the entry rate. (more: age distribution, ...)

(d)

If there is no option to become public, there exist only one type of firms. Then the stationary equilibrium has the same properties with that in the original problem where there are only firms without capacity constraint. The only difference between the equilibria in the two models is profit function of firms according to capacity constraint. (Also equilibrium φ^* changes.)

11. Durable goods 10/10 (a)

Let M' be the number of firms after entry. It is trivial that $M' \leq 1$ as long as $c_e > 0$. M' units of durable goods will be produced since there is no production cost. New durable goods would be consumed by those whose valuation θ is high. Then, the social benefit without considering entry cost is

$$\begin{aligned} R(M', s) &= \int_{1-M'}^1 \theta z d\theta + \int_{1-M'-s}^{1-M'} \theta \alpha z d\theta \\ &= \frac{1}{2}z(2M' - M'^2) + \frac{1}{2}\alpha z [2s(1 - M') - s^2] \end{aligned}$$

if $M' + s \leq 1$, and

$$\begin{aligned} R(M', s) &= \int_{1-M'}^1 \theta z d\theta + \int_0^{1-M'} \theta \alpha z d\theta \\ &= \frac{1}{2}z(2M' - M'^2) + \frac{1}{2}\alpha z(1 - M')^2 \end{aligned}$$

if $M' + s \geq 1$. Given this, the value function of social planner's problem is

$$V(M, s) = \max_e \left\{ R(M + e, s) - ec_e + \delta V\left([1 - \lambda][M + e], M + e\right) \right\}$$

(b)

FOC writes as

$$R_1(M + e^*, s) - c_e + (1 - \lambda)\delta V_1\left([1 - \lambda][M + e^*], M + e^*\right) + \delta V_2\left([1 - \lambda][M + e^*], M + e^*\right) = 0$$

where $e^* = e(M, s)$ denotes optimal entry. Envelope theorem yields

$$V_1(M, s) = R_1(M + e^*, s) + (1 - \lambda)\delta V_1\left([1 - \lambda][M + e^*], M + e^*\right) + \delta V_2\left([1 - \lambda][M + e^*], M + e^*\right)$$

$$V_2(M, s) = R_2(M + e^*, s)$$

Recall from the definition of R that when $M' + s \leq 1$,

$$R_1(M', s) = z(1 - M' - \alpha s) \geq 0$$

$$R_2(M', s) = \alpha z(1 - M' - s) \geq 0$$

and also when $M' + s \geq 1$,

$$R_1(M', s) = (1 - \alpha)z(1 - M') \geq 0$$

$$R_2(M', s) = 0$$

So in any case,

$$R_{11} < 0, R_{12} \leq 0 \text{ and } R_{22} \leq 0$$

Using FOC, the first envelope condition is equivalent to

$$V_1(M, s) = c_e$$

So FOC can be rewritten as

$$R_1(M + e^*, s) + \delta V_2\left([1 - \lambda][M + e^*], M + e^*\right) = [1 - (1 - \lambda)\delta]c_e \quad (8)$$

Also differentiating $V_1(M, s)$ with respect to s yields

$$V_{12}(M, s) = 0$$

We want to show that $e(M, s)$ is decreasing in both M and s . First, differentiate $V_2(M, s)$ with respect to M .

$$V_{12}(M, s) = (1 + e_1)R_{12}(M + e^*, s)$$

For this to be 0, it should be that $1 + e_1 = 0$, and thus $e_1 < 0$. Now suppose that $e_2 > 0$. We have $V_{22} = R_{22} \leq 0$. Let s increase with M being fixed. Then, the first term in LHS of (8) strictly decreases, since e^* increases and $R_{11} < 0$. The second term weakly decreases, but RHS does not change, which is a contradiction. So $e(M, s)$ is decreasing in both M and s .

This property of policy function implies that the number of firms is mean reverting. In other words, if there are too many firms, entry will be less and if there are too few firms, entry will be large. Accordingly the sales of the new durable good would converge to some number.

(c)

Since those who have high valuation θ consume new durable good, all consumers with valuation higher than $1 - M - e$ would prefer new, but others would prefer used. So one with valuation $1 - M - e$ is indifferent between two. Denote prices of new good and used good by p_n and p_u , respectively.

$$(1 - M - e)z - p_n + \delta p'_u = (1 - M - e)\alpha z - p_u$$

Also, one with valuation $1 - M - e - s$ should be indifferent between buying used and buying nothing.

$$(1 - M - e - s)\alpha z - p_u = 0$$

These yield

$$\begin{aligned} p_u &= (1 - M - e - s)\alpha z \\ p_n &= (1 - M - e)z - (1 - M - e)\alpha z + p_u + \delta p'_u \\ &= (1 - M - e)z - s\alpha z + \delta(1 - M' - e' - s')\alpha z \end{aligned}$$

(d)

When there is a production cost varying with time spent in the industry, all of the producing firms have low cost and get positive profit. Since λ portion of existing firms exit, in the stationary equilibrium, the same amount of new firms would enter with high cost of production. These firms may not produce in the first several periods, but after that, they would produce a good if they still survive.

12. Learning by doing 7/10 (a)

Note that firm's profit is always greater than 0 due to the given form of cost function. So we do not have to consider the outside option if $V_0 = 0$. A firm with s has the following dynamic programming problem.

$$V^p(s) = \max_q \left\{ pq - \frac{(q - \alpha s)^2}{2} + \beta V([1 - \delta]s + \lambda q) \right\}$$

This can be written in the following form as well.

$$V^p(s) = \max_{s'} \left\{ p \cdot \frac{s' - (1 - \delta)s}{\lambda} - \frac{[s' - (1 - \delta + \alpha\lambda)s]^2}{2\lambda^2} + \beta V(s') \right\}$$

A firm considering whether to enter or not has $s = 0$. This would have a linear policy function, because a return function is quadratic in the state variable s . Guess $V(s) = A_0 + A_1s + A_2s^2$, and use the method of undetermined coefficient, and then we could get the exact value function in quadratic form, and a linear policy function.

(b)

A firm with s has the following problem.

$$V(s) = \max \left\{ V_0, \max_q \left\{ pq - \frac{(q - \alpha s)^2}{2} + \beta V([1 - \delta]s + \lambda q) \right\} \right\} = \max\{V_0, V^p(s)\} \quad (9)$$

(Need to take expectation with respect to V_0 . The above V is defined as a value function after V_0 is realized. Then, since V_0 in the next period is not realized, we need to take expectation of the future value with respect to V_0 . Or we can define a value function as the one before realization of V_0 . In this case, we need to take expectation of the whole expression with respect to V_0 .) Given that a firm decides to stay in the industry, the optimal output does not change. (It does change. The value function will now be different and it will have less incentive to invest because of the likelihood of exit.) However, some firms would stop producing and exit because of high outside value. This is not likely to occur for the firm with high s , since it has a cost low enough to get a high profit from producing. In other words, firms would exit mostly in their early age, and as they spend more time in the industry, the probability that they exit decreases.

(c)

Let $\mu(s)$ be a measure of firms' experience. Note that given p , μ does not affect individual firm's decision on production and exit since " q depends only on p and s ", and exit decision depends only on V_0 and $V^p(s)$. As seen in (9), a firm with s stays in the industry with probability

$$\Pr(\text{stay}|s) = \Pr(V_0 < V^p(s))$$

An industry equilibrium is, therefore, p^* that satisfies

$$D \left(\int_0^1 q(p^*, s) \mu(ds) \right) = p^* \quad (10)$$

where $q(p, s)$ is an output policy function defined as

$$q(p, s) = \begin{cases} 0 & \text{with probability } 1 - \Pr(\text{stay}|s) \\ \text{"}a_0 + a_1p + bs\text{" solving } V^p(s) & \text{with probability } \Pr(\text{stay}|s) \end{cases}$$

We can see that under appropriate assumption there is a unique industry equilibrium p^* solving (10). This assumption includes that D is strictly decreasing. Now consider a stationary equilibrium. For an entrant to be indifferent between entering and staying out,

$$V^p(0) \Pr(\text{stay}|0) = c_e \quad (11)$$

(When you enter you also get the option of drawing an outside value V_0 . So we need to change the entry condition as

$$c_e = E[V(0)] = V^p(0) \Pr(\text{stay}|0) + E[V_0|V_0 > V^p(0)]$$

which now includes expected outside value. This is equivalent to

$$c_e = V^p(0)\Phi\left(\frac{V^p(0) - \bar{x}}{\sigma}\right) + \int_{\frac{V^p(0) - \bar{x}}{\sigma}}^{\infty} (\sigma u + \bar{x})\Phi(du)$$

where Φ is a cdf of the standard normal distribution.) Note that the above equation defines p^* . For μ to be invariant, it should be satisfied that

$$\mu(s') = \mu(s) \Pr(\text{stay}|s) \quad \text{where } s' = (1 - \delta)s + \lambda(a + bs) \quad (12)$$

recursively from $s = 0$, and $\mu(s) = 0$ for any other s . A stationary equilibrium is (p^*, μ) that satisfies equations (10), (11), and (12).

There is a unique stationary equilibrium. First, it is clear that there is only one p^* solving (11) since $V^p(0)$ is strictly increasing in p . From (12), the whole μ function can be expressed in terms of $\mu(0)$. Note that $\Pr(\text{stay}|s) < 1$ for any s , since V_0 follows a normal distribution. So the number of firms with high s is less than that of firms with low s . Note also that s is bounded above. Therefore, there exists a unique $\mu(0)$ satisfying (10).

(d) ?

As \bar{x} increases, a higher outside value is likely to be drawn. For enough number of firms to stay in the industry, price has to be high enough. (Equilibrium price will be lower.) Also since the probability that a firm exits becomes bigger, the age of firms in the industry would be such that there are more young firms and less old firms. To see this with equations, suppose that p^* is fixed. For any value of s , more firms would exit than before. In other words, $\Pr(\text{stay}|s)$ decreases by far, especially for lower s . So for (11) to hold, p^* has to increase. (Your problem is that (11) is not correct. Suppose p^* does not change. $V^p(0)$ also does not change, so as \bar{x} increases, the RHS of the new entry condition increases. So p^* has to decrease so that $V^p(0)$ decreases.) This makes $V^p(s)$ increase for any value of s , but especially for higher s . Therefore, young firms exit with much larger probability, but old firms with a little bigger probability than before. This implies that the number of young firms should be very large for μ to be invariant. The total number of firms is ambiguous.

(e)

In the stationary equilibrium, a measure of firms' experience μ predicts that a small number of large firms exist, while there are many small firms. "Also since $V^p(s)$ is an increasing function of s , large firms rarely exit, but small firms exit with high probability, which makes sense." But this result crucially depends on the assumption that V_0 is identically distributed for all firms. It is not reasonable that all firms have the same distribution of outside value. (Good point.) It is likely that a large firm has a bigger outside value than a small firm in the real world.

14. 6/10 (a)

I assume that there is no cost of production. The value function is

$$V(s) = \max \left\{ O, ps + \delta \left[(1 - \lambda)V(s) + \lambda \int_0^1 V(z)G(dz) \right] \right\}$$

It is easy to incorporate production cost into the value function. Now the value of entry should be 0, so we have

$$V^e = \int_0^1 V(s)G(ds) - c_e = 0$$

Here I implicitly assume that an outside firm does not have an outside option O . Plug this back into the value function, then

$$V(s) = \max \left\{ O, ps + \delta [(1 - \lambda)V(s) + \lambda c_e] \right\}$$

So when $V(s) > O$,

$$V(s) = ps + \delta [(1 - \lambda)V(s) + \lambda c_e]$$

or equivalently,

$$V(s) = \frac{ps + \delta \lambda c_e}{1 - \delta(1 - \lambda)}$$

Therefore we can write

$$V(s) = \max \left\{ O, \frac{ps + \delta \lambda c_e}{1 - \delta(1 - \lambda)} \right\}$$

(b)

A firm exits if and only if

$$O > \frac{ps + \delta \lambda c_e}{1 - \delta(1 - \lambda)}$$

This condition is equivalent to

$$s < \frac{1}{p} \left(O[1 - \delta(1 - \lambda)] - \delta \lambda c_e \right) \equiv s^* \quad (13)$$

(Just need to assume $O[1 - \delta(1 - \lambda)] - \delta \lambda c_e > 0$. This is the sufficient condition that is required. If this holds, $s^* > 0$ is guaranteed, so there would be exit.) s^* denotes a threshold under which a firm exits. Note here that in some cases s^* can be less than 0, so the actual threshold would be 0, and also that in other cases s^* might be greater than 1, so the actual threshold becomes 1.

(c)

Differentiating s^* with respect to λ and δ , respectively, we get

$$\begin{aligned}\frac{ds^*}{d\lambda} &= \frac{\delta}{p}(O - c_e) \leq 0 \\ \frac{ds^*}{d\delta} &= \frac{1}{p}(-O[1 - \lambda] - \lambda c_e) \leq 0\end{aligned}$$

The first result comes from the following inequality. Since $V(s) \geq O$ by definition,

$$c_e = \int_0^1 V(s)G(ds) \geq \int_0^1 OG(dz) = O$$

The second result is true when $O \geq 0$ in which case $c_e \geq O \geq 0$ holds as well. Note that if $O < 0$, we have $s^* < 0$, in which case the actual threshold is 0 instead of s^* defined above. This implies that if $O < 0$, the actual threshold does not change when δ changes slightly. (Actually we can just assume that $O > 0$, from the sufficient condition we obtained in [b]).

(d)

(Need to define competitive equilibrium price as the one that satisfies $V^e = 0$.) Let μ^* be the stationary measure of firms' shock. We have

$$D \left(\int_{s^*} s \mu^*(ds) \right) = p^* \quad (14)$$

and

$$\mu^*([s^*, s]) = M \left[G(s) - G(s^*) \right] + (1 - \lambda) \mu^*([s^*, s]) + \lambda \mu^*([s^*, 1]) \left[G(s) - G(s^*) \right]$$

where the first term is entry from outside, the second is those who were in the industry and get the same shock, and the last is those who were in the industry and get new shock which falls again within the original interval. Arrange terms to get

$$\lambda \mu^*([s^*, s]) = \left[M + \lambda \mu^*([s^*, 1]) \right] \left[G(s) - G(s^*) \right]$$

Evaluating the equation at $s = 1$,

$$\lambda \mu^*([s^*, 1]) = \left[M + \lambda \mu^*([s^*, 1]) \right] [1 - G(s^*)]$$

or equivalently,

$$\mu^*([s^*, 1]) = \frac{M[1 - G(s^*)]}{\lambda G(s^*)}$$

Plug this back into the original equation, then

$$\lambda \mu^*([s^*, s]) = \left[M + \frac{M[1 - G(s^*)]}{G(s^*)} \right] \left[G(s) - G(s^*) \right]$$

Arranging terms, we get

$$\mu^*([s^*, s]) = \frac{M \left[G(s) - G(s^*) \right]}{\lambda G(s^*)} \quad (15)$$

Now let c_e increase. Suppose that s^* increases as a result. From (15), μ^* weakly decreases at every point. (Cannot tell. M is endogenous and would increase.) Consider (14). Since there are weakly less total output produced by all firms, p^* weakly increases. This contradicts to (13), since when both p^* and c_e increase, s^* cannot increase. Therefore, when c_e increases, s^* should decrease. (The stationary equilibrium is (p^*, μ^*, M) that satisfies $V^e = 0$, (14), and (15). M is endogenous, and thus we cannot use the above method to do comparative statics. More careful investigation may work. A better way to prove the claim is suggested by Jiyeon. When c_e increases, p^* has to increase so that the entry condition holds. This is not that obvious, but note here that with p^* fixed, $\int V(s)G(ds)$ would increase by less amount than c_e increases by. So from (13), s^* has to decrease.)

(e)

(The same comments would apply to this question as well. Jiyeon has the following conjecture. When λ increases, s^* always decreases, but p^* may increase or decrease. When δ increases, p^* always decreases, but s^* may increase or decrease. She uses the same logic as in the previous question.)

When λ increases, both s^* and p^* decrease. We can prove this by way of contradiction. Let λ increase. First suppose that s^* increases. By the same argument as in (d), μ^* weakly decreases at every point, and p^* weakly increases due to fall in total output. This leads to a contradiction since when both λ and p^* increase, s^* cannot increase from (13). To see this, recall from (c) that for fixed p , an increase in λ leads to a decrease in s^* , and that an increase in p^* leads to a further decrease in s^* .

Therefore, s^* decreases. Again by the same argument as in (d), μ^* weakly increases at every point, and p^* weakly decreases due to rise in total output, which completes the proof for the case when λ increases. The same logic applies to the case when δ increases. So when δ increases, both s^* and p^* decrease.

(f)

In this model, c_e , λ , and δ affect survival of a firm through s^* . (λ also has a direct effect. Prob of survival is $(1 - \lambda) + \lambda[1 - G(s^*)]$.) The result is intuitive. When cost of entry increases, it is likely that less firms enter, inducing price to increase. So a firm with low s can still get a positive expected value. If λ is big, a firm gets a new shock with large probability, so a firm with low s would expect to get a good shock in the future and thus want to stay. If δ is large, a future profit has a large present value, so a firm with low s has a value big enough to stay. But no firm grows endogenously, since shock is exogenous and lasts for long periods when λ is low. So this model does not give any prediction on growth of firm. (Expected growth given survival is

$$\frac{(1 - \lambda) \cdot 0 + \lambda \int_{s^*} \frac{s' - s}{s} G(ds')}{(1 - \lambda) + \lambda[1 - G(s^*)]}$$

This decreases with respect to s . So smaller firms have higher growth conditional on survival.)