ECON 271C homework #4

SPRING 2008

Yang, Yong Hyeon

RED texts are comments by the professor.

BLUE texts are corrected by myself.

Industry equilibrium

10. 10/10 points (a)

A firm has the following value.

$$V(s,p) = \max\Big\{0\ ,\ \pi(s,p) + \beta\Big\{\rho V(0,p) + (1-\rho)V(s,p)\Big\}\Big\}$$

Since $V(0,p) = 0$, we have

$$V(s,p) = \max\Big\{0\ ,\ \pi(s,p) + \beta(1-\rho)V(s,p)\Big\}$$

or

$$V(s,p) = \max\left\{0\ ,\ \frac{\pi(s,p)}{1-\beta(1-\rho)}\right\}$$

Therefore, a firm with $s < s^*$ exits where s^* is defined as

$$0 = \pi(s^*, p) \tag{1}$$

An entrant has the following value.

$$V^e(p) = \int_0^1 V(s,p)G(ds) - c_e$$

An equilibrium price p^* is defined by

$$V^e(p^*) = 0 \tag{2}$$

For market clearing,

$$p^* = D\left(\int_0^1 q(s,p^*)\mu(ds)\right) \tag{3}$$

and the measure should be invariant, so

$$\mu([s^*,s]) = (1-\rho)\mu([s^*,s]) + \lambda\left[G(s) - G(s^*)\right]$$

or equivalently,

$$\mu([s^*,s]) = \frac{\lambda}{\rho}\left[G(s) - G(s^*)\right] \tag{4}$$

A stationary equilibrium is (s^*, p^*, μ, λ) that satisfies equations (1) to (4). Let $M = \mu([s^*,1])$ be the total mass of firms. The rate of turnover is the same with the exit rate, which is

$$\frac{\rho M + \lambda G(s^*)}{M} = \frac{\lambda\left[1 - G(s^*)\right] + \lambda G(s^*)}{\frac{\lambda}{\rho}\left[1 - G(s^*)\right]} = \frac{\rho}{1 - G(s^*)}$$

It is increasing in ρ.

(b) i.

Now the condition for market clearing changes as

$$p^* + \tau = D\left(\int_0^1 q(s,p^*)\mu(ds)\right) \tag{5}$$

A stationary equilibrium in the τ tax economy is (s^*, p^*, μ, λ) that satisfies equations (1), (2), (4), and (5). Note that p^* is defined by (2), so it is the same with p^* in the no tax economy. Also s^* is defined by (1), so it is the same with s^* in the no tax economy. Only μ and λ change. As we can see from (4), μ will scale down with the same rate as λ, which decreases from (5). So the absolute size of entry and exit would decrease, but the rate of entry and exit does not change.

ii.

As a sales tax τ is imposed, price would stay at the same level in the long run. But in the transition, price may decrease sharply and increase gradually. What happens is as follows. When a tax is imposed, the demand decreases. After ρ portion of firms get $s = 0$ and exit, the supply by the remaining firms may be bigger than the demand. Then the price would go down and more firms with low s would exit. There is no entry in this period. From next periods, ρ portion of existing firms exit and price goes up until it reaches p^*. As soon as the price hits p^*, there would be entry so that the supply meets the demand at p^*. Note that this happens only when τ is large enough, since otherwise the decrease in the demand is too small in the first period, so the adjustment would be immediate by some positive entry.

(c)

We have the following.

$$V_\tau(s,p) = \max\left\{0\ ,\ (1-\tau)\pi(s,p) + \beta\left\{\rho V_\tau(0,p) + (1-\rho)V_\tau(s,p)\right\}\right\}$$

So

$$V_\tau(s,p) = \max\left\{0\ ,\ \frac{(1-\tau)\pi(s,p)}{1-\beta(1-\rho)}\right\} = (1-\tau)V(s,p)$$

Therefore (1) still holds. p^* satisfies

$$0 = V_\tau^e(p^*) = \int_0^1 V_\tau(s,p^*)G(ds) - c_e = (1-\tau)\int_0^1 V(s,p^*)G(ds) - c_e$$

Since V_τ is less steeper than V, so we should have p^* higher and s^* less than before, so that the above condition holds. Combining (3) and (4), we have

$$p^* = D\left(\frac{\lambda}{\rho}\int_{s^*}^1 q(s,p^*)G(ds)\right)$$

Note that the optimal output $q(s,p)$ is not affected by profit tax. Also $q(s,p)$ is increasing in p and s. Since p^* becomes higher and s^* becomes less, λ should be less than before. Accordingly, from (4), the density of μ would be higher at $s^* < s < s^*_{old}$ and lower at $s > s^*_{old}$ than before.

17. *Variant of Klette and Kortum* **9/10** **(a)**

In terms of flow of customers, new entrants and investing firms steal $\varepsilon + \lambda\mu_1$ mass of customers from other firms. All the firms are losing one of their customers with Poisson rate η per customer, so $\eta(\mu_1 + 2\mu_2)$ customers are stolen. They should be the same. Taking $\mu_1 + 2\mu_2 = 1$ into account, we have

$$\eta = \varepsilon + \lambda\mu_1 \tag{6}$$

(b)

Let v_1 and v_2 be the value of firms with one customer and two customers, respectively. Since a firms with no customer should exit and get a value of 0, that is, $v_0 = 0$,

$$rv_1 = \overline{\pi} - \eta v_1 + \max_{\lambda}\Big[\lambda(v_2 - v_1) - c(\lambda)\Big] \tag{7}$$

$$rv_2 = 2\overline{\pi} - \eta(v_2 - v_1) \tag{8}$$

An entrant has the following value.

$$v^e = v_1 - F$$

It should be the case that $v^e \leq 0$ with equality if $\varepsilon > 0$. This can be written as

$$v_1 - F \leq 0 \quad \text{with} \quad \varepsilon(v_1 - F) = 0 \tag{9}$$

For the measure of firms to be invariant, the inflow and the outflow of each type of firms should be the same. Hence,

$$\varepsilon + 2\eta\mu_2 = \lambda\mu_1 + \eta\mu_1$$

$$\lambda\mu_1 = 2\eta\mu_2$$

where the first is for firms with one customer and the second is for firms with two customers. Rewrite the first equation using the second equation, then

$$\varepsilon = \eta\mu_1 \tag{10}$$

$$\lambda\mu_1 = 2\eta\mu_2 \tag{11}$$

A stationary equilibrium is $(v_1, v_2, \lambda, \eta, \varepsilon, \mu_1, \mu_2)$ that satisfies the maximization problem in (7) and equations (6) to (11). There are 7 unknowns and 7 equations, so the equilibrium is well defined.

(c)

v_1 is defined by (9). Since $\varepsilon > 0$, we have $v_1 = F$. Also, from the first order condition of the maximization problem in (7),

$$v_2 - v_1 - \lambda = 0$$

or equivalently,

$$\lambda = v_2 - v_1$$

Using these, (7) and (8) can be written as

$$rF = \overline{\pi} - \eta F + \frac{1}{2}\lambda^2$$
$$r(\lambda + F) = 2\overline{\pi} - \eta\lambda$$

From the above equations, λ and η are uniquely defined. v_2 is determined so that $v_2 = v_1 + \lambda$. Then, (6), (10) and (11) characterize ε, μ_1 and μ_2 uniquely.

(d)
Use $\mu_1 + 2\mu_2 = 1$ and (11) to write

$$\lambda\mu_1 = \eta(1 - \mu_1)$$

Rearranging,

$$\mu_1 = \frac{\eta}{\eta + \lambda}$$

Use (11) again to obtain

$$\mu_2 = \frac{\lambda}{2(\eta + \lambda)}$$

(e)
Suppose F is very close to 0. (If $F \to 0$ and $r \to 0$, $\mu_1 \to \frac{1}{2}$, and $c(\lambda) \to \overline{\pi}$ so $R \to \frac{\overline{\pi}}{2}$.) Then, there would be many entrants, so there are many type 1 firms. This implies that the number of type 2 firms is small, and that λ is small. So there is low total investment $R = \mu_1 c(\lambda)$. (This is ambiguous. λ is small, but μ_1 is large.) "As F increases, the number of entrants decreases, so μ_1 becomes small but μ_2 becomes large. Many type 1 firms would invest more, so λ increases. Up to some point, λ increases fast, and μ_1 decreases slowly, which means that R increases. But in the end, if F is very high, entry size is too small, and μ_1 is very small, so there would be low R. Therefore R would decrease as F increases." (?)

Industry equilibrium: Locations
23. **5/10** You have not interpreted this problem correctly and unfortunately this trivializes the answer. **(a)**
A value of a firm in location l with a shock φ is

$$V_l(\varphi, p_l) = \max\left\{0\ ,\ \pi(\varphi, p_l) + \beta \int V_l(\varphi', p_l) F(d\varphi'|\varphi)\right\}$$

An exit threshold φ_l^* is defined as

$$0 = \pi(\varphi_l^*, p_l) + \beta \int V_l(\varphi', p_l) F(d\varphi'|\varphi_l^*) \tag{12}$$

An entrant to location l has the following value. (Firm receives 2 shocks at entry and picks the more value between the two markets which it may enter.)

$$V_l^e(p_l) = \int V_l(\varphi, p_l) G(d\varphi) - c_e$$

p_l is defined so that

$$V_l^e(p_l) = 0 \tag{13}$$

The invariant measure μ_l and the entry intensity λ_l should satisfy

$$\mu_l([\varphi_l^*, s]) = \int \Big[F(s|\varphi) - F(\varphi_l^*|\varphi)\Big]\mu_l(d\varphi) + \lambda_l\Big[G(s) - G(\varphi_l^*)\Big] \tag{14}$$

$$D_l(p_l) = \int q(\varphi, p_l)\mu_l(d\varphi) \tag{15}$$

$(G(s) - G(\varphi_l^*)$ in (14) is not the right distribution of entrant's shock. Must condition on $\varphi_l \geq \varphi_l'$. Must condition on $\varphi_l \geq \varphi_r$? What is φ_l'?) Equations (12) to (15) define a stationary equilibrium $(\varphi_l^*, p_l, \mu_l, \lambda_l)$ in location l. Now condiser location r. A value of a firm in location r with a shock φ is

$$V_r(\varphi, p_r) = \max\Big\{0\ ,\ \pi(\varphi, p_r) + \beta\int V_r(\varphi', p_r)F(d\varphi'|\varphi)\Big\}$$

An exit threshold φ_r^* is defined as

$$0 = \pi(\varphi_r^*, p_r) + \beta\int V_r(\varphi', p_r)F(d\varphi'|\varphi_r^*) \tag{16}$$

p_r is defined so that an entrant to location r has a value of 0.

$$0 = V_r^e(p_r) = \int V_r(\varphi, p_r)G(d\varphi) - c_e \tag{17}$$

The invariant measure μ_r and the entry intensity λ_r should satisfy

$$\mu_r([\varphi_r^*, s]) = \int \Big[F(s|\varphi) - F(\varphi_r^*|\varphi)\Big]\mu_r(d\varphi) + \lambda_r\Big[G(s) - G(\varphi_r^*)\Big] \tag{18}$$

$$D_r(p_r) = \int q(\varphi, p_r)\mu_r(d\varphi) \tag{19}$$

Equations (16) to (19) define a stationary equilibrium $(\varphi_r^*, p_r, \mu_r, \lambda_r)$ in location r. ($\times$)

(b)

Note that π and F are in common, so that $V_l(\varphi, p) = V_r(\varphi, p)$ from definition of value functions. Also G and c_e are in common. Therefore, entry conditions in both locations imply that $p_l = p_r$. So from exit conditions, $\varphi_l^* = \varphi_r^*$. Since prices, exit thresholds, value functions and transition distributions are the same, the expected lifetime of firms would be the same in both locations. So the rate of turnover is also the same. To see this from the exit rate, consider the invariant measure conditions. Since $D_l(p) = \gamma D_r(p)$, we can infer that

$$\lambda_l = \gamma\lambda_r \quad \text{and} \quad \mu_l([\varphi^*, s]) = \gamma\mu_r([\varphi^*, s]) \;\; \text{for all } s$$

Therefore the exit rate is the same in both locations as can be seen in the following.

$$\frac{\int F(\varphi^*|\varphi)\mu_l(d\varphi) + \lambda_l G(\varphi^*)}{\mu_l([\varphi^*, \infty])} = \frac{\int F(\varphi^*|\varphi)\mu_r(d\varphi) + \lambda_r G(\varphi^*)}{\mu_r([\varphi^*, \infty])}$$

This implies that the rate of turnover is the same in both locations. The average size of firms would also be the same since $\mu_l([\varphi^*, s]) = \gamma\mu_r([\varphi^*, s])$ for all s. So this model is not consistent with the observation that the average size of firms is larger in larger markets.

(c)

Let $V^2(\varphi, p_l, p_r)$ be the value of the firm that operates in both locations. (Shocks in both locations should not be the same.) Then,

$$V^2(\varphi, p_l, p_r) = V_l(\varphi, p_l) + V_r(\varphi, p_r) = \max\left\{0\ ,\ \pi(\varphi, p_l) + \pi(\varphi, p_r) + \beta\int V^2(\varphi', p_l, p_r)F(d\varphi'|\varphi)\right\}$$

where V_l and V_r are defined as in (a). Let c_e^s be an entry cost into a second market. The incumbent in location l solves the following dynamic programming problem before deciding to enter into r.

$$V_l^1(\varphi, p_l, p_r) = \max\left\{0\ ,\ V^2(\varphi, p_l, p_r) - c_e^s\ ,\ \pi(\varphi, p_l) + \beta\int V_l^1(\varphi', p_l, p_r)F(d\varphi'|\varphi)\right\}$$

The incumbent in location l enters into r if

$$V^2(\varphi, p_l, p_r) - c_e^s \geq \pi(\varphi, p_l) + \beta\int V_l^1(\varphi', p_l, p_r)F(d\varphi'|\varphi)$$

There is a threshold φ_l^s such that if $\varphi \geq \varphi_l^s$, the incumbent in l enters into r. Also let φ^* be an exit threshold of firms operating in one location. Firms that originally start in r initially draw $\varphi \geq \varphi^*$ since otherwise they would have exited immediately. Since $F(\varphi'|\varphi)$ is decreasing in φ, firms that enter into a second market r are more likely to survive than firms that originally start in r if $\varphi_l^s > \varphi^*$. Compare the following conditions that φ_l^s and φ^* satisfy.

$$\begin{aligned} V^2(\varphi_l^s, p_l, p_r) - c_e^s = \pi(\varphi_l^s, p_l) + \beta\int V_l^1(\varphi', p_l, p_r)F(d\varphi'|\varphi_l^s) > 0 \\ 0 = \pi(\varphi^*, p_l) + \beta\int V_l^1(\varphi', p_l, p_r)F(d\varphi'|\varphi^*) > V^2(\varphi^*, p_l, p_r) - c_e^s \end{aligned}$$

So c_e^s should be big enough so that $V^2(\varphi^*, p_l, p_r) - c_e^s < 0$ at an exit threshold φ^*. Also as φ goes up, $V^2(\varphi, p_l, p_r)$ should increase faster than $\pi(\varphi, p_l) + \beta\int V_l^1(\varphi', p_l, p_r)F(d\varphi'|\varphi)$. This requires that

$$V^2(\varphi, p_l, p_r) - \pi(\varphi, p_l) - \beta\int V_l^1(\varphi', p_l, p_r)F(d\varphi'|\varphi)$$

is increasing in φ. Using the definition of V^2, we can rewrite this as

$$\pi_1(\varphi, p_r) + \beta\int\left[V^2(\varphi', p_l, p_r) - V_l^1(\varphi', p_l, p_r)\right]F_2(d\varphi'|\varphi) \geq 0$$

This is satisfied when $V^2 - V_l^1$ is an increasing function of φ', since $F(\varphi'|\varphi)$ is decreasing in φ.

(d)

In the model (c), an entrant from outside into l has a value

$$V_l^e(p_l) = \int V_l^1(\varphi, p_l, p_r)G(d\varphi) - c_e$$

Since V_l^1 includes the option to enter the other market in the future, it is steeper than V_l in (a). So the equilibrium p_l would be lower and the threshold φ^* would be higher than in (a). Because of symmetry, prices and exit thresholds would be the same in both locations. φ^s is greater than φ^* as shown in (c). There is another threshold φ^{**} for exit of firms operating in both locations.

24. 9/10 (a)

Since a firm gets $\pi(n,s,p)$ in each market, its one period profit is $n\pi(n,s,p)$ if it operates in n markets. The value function would be the following. For $n = 1, \cdots, N-1$,

$$\begin{aligned} v(n,s,p) = \int \max \Big\{ &(n+1)\pi(n+1,s,p) - c + \beta \int v(n+1,s',p)F(ds'|s) \ , \\ &n\pi(n,s,p) + \beta \int v(n,s',p)F(ds'|s) \ , \\ &(n-1)\pi(n-1,s,p) + \beta \int v(n-1,s',p)F(ds'|s) \Big\} H(dc) \end{aligned}$$

where conventionally $\pi(0,s,p) = v(0,s,p) = 0$. When $n = N$,

$$\begin{aligned} v(N,s,p) = \max \Big\{ &N\pi(N,s,p) + \beta \int v(N,s',p)F(ds'|s) \ , \\ &(N-1)\pi(N-1,s,p) + \beta \int v(N-1,s',p)F(ds'|s) \Big\} \end{aligned}$$

There is no fresh entry in the stationary equilibrium if there is no exit from the industry. So we need an appropriate assumption to ensure that there is fresh entry in the stationary equilibrium. For some firms to exit from the industry, it should be possible that a firm operating in one market exits from the market when it gets a really bad shock. In other words, for any p, there should exist s and c such that

$$\pi(1,s,p) + \beta \int v(1,s',p)F(ds'|s) < 0$$

$$2\pi(2,s,p) - c + \beta \int v(2,s',p)F(ds'|s) < 0$$

If we have $\inf_s \pi(1,s,p) < 0$, $\inf_s \pi(2,s,p) < 0$ and for any s', $\lim_{s \to -\infty} F(s'|s) = 1$, there is a stationary equilibrium with fresh entry every period.

(b)

There are thresholds for entering additional market, $c_1(s), \cdots, c_{N-1}(s)$. If a firm with n and s draws $c < c_n(s)$, it pays c and enters additional market. They are the maximum value among c satisfying both of the following conditions.

$$(n+1)\pi(n+1,s,p) - c + \beta \int v(n+1,s',p)F(ds'|s) \geq n\pi(n,s,p) + \beta \int v(n,s',p)F(ds'|s)$$

$$\begin{aligned} &(n+1)\pi(n+1,s,p) - c + \beta \int v(n+1,s',p)F(ds'|s) \geq \\ &\qquad (n-1)\pi(n-1,s,p) + \beta \int v(n-1,s',p)F(ds'|s) \end{aligned}$$

Also there are thresholds for exiting a market, $s_1, \cdots, s_N$. If a firm with n has $s < s_n$ but draws $c > c_n(s)$, it exits a market and operates only in $n-1$ markets. (How do you know it is a threshold?) They satisfy

$$(n-1)\pi(n-1, s, p) + \beta \int v(n-1, s', p)F(ds'|s) = n\pi(n, s, p) + \beta \int v(n, s', p)F(ds'|s)$$

An entrant would have the following value.

$$v^e(p) = \int v(1, s, p)G(ds) - c_e$$

An equilibrium p would satisfy $v^e(p) = 0$ when there are exit and entry. The invariant measure of firms, $\mu_1, \cdots, \mu_N$ would satisfy the following condition. For $n = 2, \cdots, N-1$,

$$\begin{aligned}
\mu_n([-\infty, s]) = & \int \max\Big\{0, F(s|z) - F(s_n|z)\Big\}\Big[1 - H\big(c_n(z)\big)\Big]\mu_n(dz) && \text{(stay)} \\
& + \int F(\min\{s, s_{n+1}\}|z)\Big[1 - H\big(c_{n+1}(z)\big)\Big]\mu_{n+1}(dz) && \text{(exit)} \\
& + \int F(s|z)H\big(c_{n-1}(z)\big)\mu_{n-1}(dz) && \text{(enter)}
\end{aligned}$$

For $n = 1$,

$$\begin{aligned}
\mu_1([-\infty, s]) = & \int \max\Big\{0, F(s|z) - F(s_1|z)\Big\}\Big[1 - H\big(c_1(z)\big)\Big]\mu_1(dz) && \text{(stay)} \\
& + \int F(\min\{s, s_2\}|z)\Big[1 - H\big(c_2(z)\big)\Big]\mu_2(dz) && \text{(exit)} \\
& + \lambda \max\Big\{0, G(s) - G(s_1)\Big\} && \text{(enter)}
\end{aligned}$$

For $n = N$,

$$\begin{aligned}
\mu_N([-\infty, s]) = & \int \max\Big\{0, F(s|z) - F(s_N|z)\Big\}\mu_N(dz) && \text{(stay)} \\
& + \int F(s|z)H\big(c_{N-1}(z)\big)\mu_{N-1}(dz) && \text{(enter)}
\end{aligned}$$

For market clearing,

$$p = D\left(\sum_{n=1}^{N} \int \frac{n}{N} q(n, s, p)\mu_n(ds)\right)$$

where $q(n, s, p)$ is the optimal output of a firm in each market. Note that the total output of all firms is $\sum_n \int nq(n, s, p)\mu_n(ds)$, so in each market, $\frac{1}{N}\sum_n \int nq(n, s, p)\mu_n(ds)$ units of output are supplied. A stationary equilibrium is $(p, s_1, \cdots, s_N, c_1(s), \cdots, c_{N-1}(s), \mu_1, \cdots, \mu_N, \lambda)$ that satisfies the above conditions.

(c)

If we drop the assumption that firms can exit at most one market in a given period, then this model predicts that the smallest size of firms participating in more markets be bigger. "Even if we maintain the assumption, the average size of firms operating in more markets would be bigger according to the model." (Not obvious.)

(d)

This model predicts that the more markets firms are participating in, the older the firms are likely to be on average, since it takes longer to enter more markets. (Also selection effects should be considered.)