2000 Fall 1. Radially Parallel Preferences

(a) Let $U(c) > U(c')$. By continuity and monotonicity of preferences, there exist $\lambda < 1$ such that $U(c') = U(\lambda c)$. By radial parallelness, $U(\theta c') = U(\theta\lambda c) < U(\theta c)$.

(b) Since $c(p, I)$ is feasible under I, $\theta c(p, I)$ is feasible under θI. By definition of $c(\cdot, \cdot)$,

$$U(\theta c(p, I)) \leq U(c(p, \theta I)) \tag{1}$$

Again, $\frac{1}{\theta}c(p, \theta I)$ is feasible under I, so

$$U\left(\frac{1}{\theta}c(p, \theta I)\right) \leq U(c(p, I))$$

By radial parallelness,

$$U(c(p, \theta I)) \leq U(\theta c(p, I)) \tag{2}$$

Thus (1) and (2) together imply $U(c(p, \theta I)) = U(\theta c(p, I))$, or $c(p, \theta I) = \theta c(p, I)$

(c) Note that $V(c) = c^{1-\sigma}/(1-\sigma)$. Suppose $U(c) = U(c')$. Then,

$$U(c) = \sum_{i=1}^{n} \alpha_i \frac{1}{1-\sigma} c^{1-\sigma} = \sum_{i=1}^{n} \alpha_i \frac{1}{1-\sigma} c'^{1-\sigma} = U(c')$$

Multiplying $\theta^{1-\sigma}$ on both sides,

$$U(\theta c) = \sum_{i=1}^{n} \alpha_i \frac{1}{1-\sigma} \theta^{1-\sigma} c^{1-\sigma} = \sum_{i=1}^{n} \alpha_i \frac{1}{1-\sigma} \theta^{1-\sigma} c'^{1-\sigma} = U(\theta c')$$

(d) When preferences are radially parallel, all individuals consume a fraction of aggregate endowment. So we can introduce a representative agent who has aggregate endowment. His utility maximization problem is

$$\max U(c) = \sum_{i=1}^{n} \alpha_i V(c_i) \qquad \text{subject to} \quad \sum_{i=1}^{n} p_i c_i = \sum_{i=1}^{n} p_i \omega_i$$

FOC reads as $\alpha_i V'(c_i) = \lambda p_i$, so using $V'(c) = c^{-\sigma}$,

$$\frac{p_{i+1}}{p_i} = \frac{\alpha_{i+1} V'(c_{i+1})}{\alpha_i V'(c_i)} = \frac{\alpha_{i+1}}{\alpha_i} \left(\frac{c_i}{c_{i+1}}\right)^{\sigma}$$

Using market clearing condition $c = \omega$, we have

$$\frac{p_{i+1}}{p_i} = \frac{\alpha_{i+1}}{\alpha_i} \left(\frac{\omega_i}{\omega_{i+1}}\right)^{\sigma}$$

(e) One period interest rate depends on the price of commodities of two adjcent periods. Since

$$\frac{p_{i+1}}{p_i} = \delta \left(\frac{\omega_i}{\omega_{i+1}}\right)^{\sigma}$$

the above price ratio is less than 1, if σ is close enough to 0. Of course, $\omega_i > \omega_{i+1}$ ensures that the above price ratio could be larger than 1 when σ is large enough.

2000 Fall 2. Short Takes

(a) True.
Let $\overline{w}$ be the wealth, q_1 be expenditure to risky asset. Then a agent's utility is

$$v(q_1) \equiv Eu(\overline{w} - q_1 + q_1(1 + r_s)) = \sum_{s=1}^{S} \pi_s u(\overline{w} - q_1 + q_1(1 + r_s))$$

$$v'(0) = \sum_{s=1}^{S} r_s u'(\overline{w}) > 0$$

Therefore, this agent consumes positive amount of risky asset $q_1 > 0$.

(b) False.
Since there is no aggregate risk, every individual can consume a fraction of aggregate endowment and diversify out whole risk. So there is no risk premium on any asset. We can see this in mathematical way. Since every individual consumes on no risk line,

$$\frac{p_s}{p_t} = \frac{\pi_s}{\pi_t} \quad \forall s, t$$

Thus a value of an asset A is

$$\sum_{s=1}^{S} p_s c_s^A = \sum_{s=1}^{S} \lambda \pi_s c_s^A = \lambda E c$$

for a certain λ. This depends only on expected value of the asset Ec.

(c) True.
This is the first law of input demand and supply. Whether a firm is a monopolist or not in the output market, it would buy less input whose price goes up (in weak sense). Actually, even if the firm is a price-setter in every other market including output market, it would buy less input whose price goes up as long as it is a price-taker in this input market. Refer to the proof of two versions of the first law.

(d) False.

$$\begin{aligned} MRTS_1 &= \left.\frac{\frac{1}{3}K_1^{\frac{2}{3}}L_1^{-\frac{2}{3}}}{\frac{2}{3}K_1^{-\frac{1}{3}}L_1^{\frac{1}{3}}}\right|_{(\overline{L},\overline{K})} = \left.\frac{K_1}{2L_1}\right|_{(\overline{L},\overline{K})} = \frac{1}{2} \\ MRTS_2 &= \left.\frac{\frac{1}{2}K_1^{\frac{1}{2}}L_1^{-\frac{1}{2}}}{\frac{1}{2}K_1^{-\frac{1}{2}}L_1^{\frac{1}{2}}}\right|_{(\overline{L},\overline{K})} = \left.\frac{K_1}{L_1}\right|_{(\overline{L},\overline{K})} = 1 \end{aligned}$$

Therefore wage-rental ratio is always in the interval $[\frac{1}{2}, 1]$.