2001 Fall 1. Constant returns to scale economy

(a) Since

$$MRTS_A|_{(200,400)} = \left.\frac{z_{2A}}{z_{1A}}\right|_{(200,400)} = MRTS_B|_{(200,400)} = 2$$

the only possible input price ratio is $r_1/r_2 = 2$.

(b) Since the cost functions to produce A and B are

$$c_A(x_A) = \left(\frac{1}{2}r_1 + r_2\right)x_A \quad \text{and} \quad c_B(x_B) = \left(\frac{1}{2\sqrt{2}}r_1 + \frac{1}{\sqrt{2}}r_2\right)x_A$$

If the demand for commodity A rises, thus the price of A is $\sqrt{2}$ times as high as that of B, then only commodity A will be produced. The input price ratio will be always

$$MRTS_A = MRTS_B = 2$$

(c) If the output price ratio is

$$\frac{p_A}{p_B} \leq \sqrt{2}$$

then commodity B will be produced. If the inequality is strict, then only B will be produced.

2001 Fall 2. Extracting a natural resource

(a) The planner will maximize

$$\max \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \int_0^{q_t} p(q)dq \quad \text{subject to} \quad \sum_{t=1}^{T} q_t \leq \bar{Q}$$

(b) Lagrangian is

$$\mathcal{L} = \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \int_0^{q_t} p(q)dq + \lambda \left(\bar{Q} - \sum_{t=1}^{T} q_t \right)$$

FOC is

$$\frac{1}{(1+r)^{t-1}} p(q_t) - \lambda = 0 \quad \forall t$$

The price ratio would be

$$\frac{p(q_{t+1})}{p(q_t)} = \frac{\lambda(1+r)^t}{\lambda(1+r)^{t-1}} = 1+r$$

(c) A monopolist will maximize

$$\max \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} q_t^m p(q_t^m) \quad \text{subject to} \quad \sum_{t=1}^{T} q_t^m \leq \bar{Q}$$

FOC is

$$\frac{1}{(1+r)^{t-1}} \left[p(q_t^m) + q_t^m \cdot \frac{\partial p(q_t^m)}{\partial q_t^m} \right] - \lambda = 0 \quad \forall t$$

In general, the solution and the price ratio would be different from those in (b).

(d) $MR(q)$ is

$$MR(q) = p(q) + q \cdot \frac{\partial p(q)}{\partial q} = q^{-\frac{1}{\varepsilon}} + q \cdot \left(-\frac{1}{\varepsilon} \right) q^{-1-\frac{1}{\varepsilon}} = \left(1 - \frac{1}{\varepsilon} \right) p(q)$$

Thus FOC of a monopolist will be

$$\left(1 - \frac{1}{\varepsilon} \right) \frac{1}{(1+r)^{t-1}} p(q_t) - \lambda = 0 \quad \forall t$$

The price ratio is

$$\frac{p(q_{t+1})}{p(q_t)} = \frac{\lambda(1+r)^t}{\lambda(1+r)^{t-1}} = 1+r$$

which is the same with that in (b).

(e) If this is owned commonly (not that each agent has a fraction of the ownership for the natural resource), each agent will maximize

$$\max \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} q_t p_t$$

because the marginal cost is 0. The optimal solution for each agent is to extract q_t as much as possible, as long as p_t is greater than 0. This is much different from a social optimal solution and worse than the outcome in a monopolist case. This is called tragedy of the common. If each agent shares a fraction of the whole profit, the outcome would be the same with the social optimum.