2001 Fall 1. Constant returns to scale economy

(a) Since
\[ \frac{2A}{2A} \bigg|_{(200,400)} = \frac{2A}{2A} \bigg|_{(200,400)} = MRTS_A = MRTS_B = 2 \]
the only possible input price ratio is \( r_1/r_2 = 2 \).

(b) Since the cost functions to produce A and B are
\[ c_A(x_A) = \left( \frac{1}{2} r_1 + r_2 \right) x_A \quad \text{and} \quad c_B(x_B) = \left( \frac{1}{2\sqrt{2}} r_1 + \frac{1}{\sqrt{2}} r_2 \right) x_A \]
If the demand for commodity A rises, thus the price of A is \( \sqrt{2} \) times as high as that of B, then only commodity A will be produced. The input price ratio will be always
\[ MRTS_A = MRTS_B = 2 \]

(c) If the output price ratio is
\[ \frac{p_A}{p_B} \leq \sqrt{2} \]
then commodity B will be produced. If the inequality is strict, then only B will be produced.
2001 Fall 2. Extracting a natural resource

(a) The planner will maximize
\[
\max \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \int_{0}^{q_{t}} p(q) dq \quad \text{subject to} \quad \sum_{t=1}^{T} q_{t} \leq Q
\]

(b) Lagrangian is
\[
\mathcal{L} = \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \int_{0}^{q_{t}} p(q) dq + \lambda \left( Q - \sum_{t=1}^{T} q_{t} \right)
\]
FOC is
\[
\frac{1}{(1+r)^{t-1}} p(q_{t}) - \lambda = 0 \quad \forall t
\]
The price ratio would be
\[
\frac{p(q_{t+1})}{p(q_{t})} = \frac{\lambda(1+r)^{t}}{\lambda(1+r)^{t-1}} = 1 + r
\]

(c) A monopolist will maximize
\[
\max \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} q_{m}^{m} p(q_{m}^{m}) \quad \text{subject to} \quad \sum_{t=1}^{T} q_{m}^{m} \leq Q
\]
FOC is
\[
\frac{1}{(1+r)^{t-1}} \left[ p(q_{m}^{m}) + q_{m}^{m} \cdot \frac{\partial p(q_{m}^{m})}{\partial q_{m}^{m}} \right] - \lambda = 0 \quad \forall t
\]
In general, the solution and the price ratio would be different from those in (b).

(d) MR(q) is
\[
MR(q) = p(q) + q \cdot \frac{\partial p(q)}{\partial q} = q^{-\frac{1}{\varepsilon}} + q \cdot \left( -\frac{1}{\varepsilon} \right) q^{-1-\frac{1}{\varepsilon}} = \left( 1 - \frac{1}{\varepsilon} \right) p(q)
\]
Thus FOC of a monopolist will be
\[
\left( 1 - \frac{1}{\varepsilon} \right) \frac{1}{(1+r)^{t-1}} p(q_{t}) - \lambda = 0 \quad \forall t
\]
The price ratio is
\[
\frac{p(q_{t+1})}{p(q_{t})} = \frac{\lambda(1+r)^{t}}{\lambda(1+r)^{t-1}} = 1 + r
\]
which is the same with that in (b).

(e) If this is owned commonly (not that each agent has a fraction of the ownership for the natural resource), each agent will maximize
\[
\max \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} q_{t} p_{t}
\]
because the marginal cost is 0. The optimal solution for each agent is to extract \( q_{t} \) as much as possible, as long as \( p_{t} \) is greater than 0. This is much different from a social optimal solution and worse than the outcome in a monopolist case. This is called tragedy of the common. If each agent shares a fraction of the whole profit, the outcome would be the same with the social optimum.