2002 Fall 1. Short Takes

I am not sure if this is really Riley problem. Probably each item might be from each course.

(a) The statement is true in terms of profits of individual firms. In Bertrand duopoly, the equilibrium price would be the marginal cost of both firms.[1] But in Cournot duopoly, the price would be usually higher than the marginal cost and close to monopoly price.
The statement is false in terms of social benefit. In Bertrand duopoly, $p = MC$, which enables us to have socially efficient outcome. In Cournot competition, firms compete by varying output level, whereas in Bertrand competition, they compete by changing price level.

(b) True.
This is because in a perfectly competetive market, an individual doesn't affect the price.

(c) True.
If all agents have the same homothetic preferences, we can introduce a representative agent with the preference and the aggregate endowment, and obtain equilibrium prices easily without solving the system of equations. If the preferences are homothetic but not identical, we cannot introduce a representative agent, so we have to solve the system of eqautions. It would not be easy.

[1] When it is impossible to make both MC equal, the price would be the higher MC.

2002 Fall 2. Life Cycle Model

(a)

$$\max \sum_{t=1}^{T} \delta^{t-1} v(c_t) \quad \text{subject to} \quad \sum_{t=1}^{T}\sum_{j=1}^{n} \frac{p_{jt}c_{jt}}{(1+r)^{t-1}} \leq \sum_{t=1}^{T} \frac{y}{(1+r)^{t-1}}$$

Form a Lagrangian and obtain FOC with respect to c_{jt}

$$\delta^{t-1}\frac{\partial v}{\partial c_{tj}} - \lambda \frac{p_{jt}}{(1+r)^{t-1}} = 0$$

Since p_{jt} is constant over time t, we can get from the above FOC

$$\frac{\delta \dfrac{\partial v(c)}{\partial c_{j,t+1}}}{\dfrac{\partial v(c)}{\partial c_{jt}}} = \frac{1}{1+r} \tag{1}$$

So if

$$\frac{\partial v(c)/\partial c_{j,t+1}}{\partial v(c)/\partial c_{jt}} = \frac{1}{\delta(1+r)} < 1 \quad \forall j$$

then $c_{j,t+1} > c_{jt}$ for all j since $v(c)$ is concave. Again, since price and income are constant over time, $c_{j,t+1} > c_{jt}$ for all j implies that total expenditure increases over time. So the above condition is sufficient condition for that the individual save in the early periods and dissave in the later periods.

(b) Let W_t be an expenditure in period t such that

$$\sum_{t=1}^{T} \frac{W_t}{(1+r)^{t-1}} = \sum_{t=1}^{T} \frac{y}{(1+r)^{t-1}} \tag{2}$$

Since the utility function is Cobb-Douglas, we can easily obtain the optimal consumption within period t as

$$c_{1t} = \frac{\alpha}{\alpha+\beta}\frac{W_t}{p_{1t}} \quad , \quad c_{2t} = \frac{\beta}{\alpha+\beta}\frac{W_t}{p_{2t}} \tag{3}$$

Using (3), $p_{1,t+1} = p_{1t}(1+\theta)$, and $p_{2,t+1} = p_{2t}$, (1) reads as

$$\left(\frac{W_{t+1}}{W_t}\right)^{\alpha+\beta-1} = \frac{(1+\theta)^\alpha}{\delta(1+r)}$$

Assuming $\alpha + \beta < 1$, the optimal expenditure W_t is increasing over time if

$$\frac{(1+\theta)^\alpha}{\delta(1+r)} < 1$$

(c) Using (3), the optimization problem is equivalent to choosing W_t maximizing

$$V(W_1, \cdots, W_T) = \sum_{t=1}^{T} \delta^{t-1} c_{1t}^{\alpha} c_{2t}^{\beta} = \sum_{t=1}^{T} \delta^{t-1} \left(\frac{\alpha}{p_{1t}}\right)^{\alpha} \left(\frac{\beta}{p_2 t}\right)^{\beta} W_t = \sum_{t=1}^{T} \left[\frac{\delta}{(1+\theta)^\alpha}\right]^{t-1} \left(\frac{\alpha}{p_{11}}\right)^{\alpha} \left(\frac{\beta}{p_2 1}\right)^{\beta} W_t$$

subject to (2). Since this is linear in W_t, the individual consumes everything in the first period if

$$\frac{\delta}{(1+\theta)^\alpha} < \frac{1}{1+r} \quad \text{or equivaelntly} \quad \delta(1+r) < (1+\theta)^\alpha$$

and consumes everything in the last period if the direction of inequality is opposite.

2002 Fall 5. Benevolent Consumers

(a) If Bev doesn't give her part of income to Alex, the optimal consumption would be

$$x_a = \frac{I_a}{2p_x}, y_a = \frac{I_a}{2p_y}, x_b = \frac{I_b}{2p_x}, y_b = \frac{I_b}{2p_y}$$

At these consumption levels,

$$\frac{\partial U_b}{\partial x_b} = \frac{2p_x}{I_b} \quad , \quad \frac{\partial U_b}{\partial y_b} = \frac{2p_y}{I_b}$$

whereas

$$\frac{\partial U_b}{\partial x_a} = \alpha\frac{2p_x}{I_a} \quad , \quad \frac{\partial U_b}{\partial y_a} = \alpha\frac{2p_y}{I_a}$$

If the latter are greater than the former, Bev can increase her utility by giving her part of income to Alex. This condition is

$$\alpha I_b > I_a$$

(b) Let t be the transfer of income from Bev to Alex. Then Bev's indirect utility is

$$V(t) = \log\frac{I_b - t}{2p_x} + \log\frac{I_b - t}{2p_y} + \alpha\left(\log\frac{I_a + t}{2p_x} + \log\frac{I_a + t}{2p_y}\right)$$

Differentiating with respect to t,

$$V'(t) = -\frac{2p_x}{I_b - t} - \frac{2p_y}{I_b - t} + \frac{2p_x\alpha}{I_a + t} + \frac{2p_y\alpha}{I_a + t} = 0$$

$$t = \frac{\alpha I_b - I_a}{1 + \alpha}$$

(c) Let x_a^*, y_a^*, x_b^* and y_b^* be the optimal consumption with utility function u and income I_a and I_b. Bev will give part of her income to Alex if and only if

$$\alpha\frac{\partial u(x_a^*, y_a^*)}{\partial x}\frac{\partial x_a^*}{\partial I_a} + \alpha\frac{\partial u(x_a^*, y_a^*)}{\partial y}\frac{\partial y_a^*}{\partial I_a} > \frac{\partial u(x_b^*, y_b^*)}{\partial x}\frac{\partial x_b^*}{\partial I_b} + \frac{\partial u(x_b^*, y_b^*)}{\partial y}\frac{\partial y_b^*}{\partial I_b}$$

Note that LHS is marginal utility of Bev by increasing I_a by 1, whereas RHS is marginal cost of Bev by decreasing I_b by 1.

(d) Without transfer, an equilibrium is the set of prices and allocations such that

(1) under prices, every agent maximizes her utility by choosing her own consumption under her income.
(2) markets clear.

This is not Pareto efficient, because the consumption of type A people creates a positive externality to the utility of type B people. But if we introduce transfer, an equilibrium would be the set of prices, transfers from type B people to type A people, and allocations such that

(1′) every A maximizes her utility by choosing her own consumption under her income and received transfer.
(2′) every B maximizes her utility by choosing transfer and her own consumption under her income.
(3′) markets clear.

This would be Pareto efficient, because all the externalities are internalized through transfer.