2003 Fall 1. Firm Scale

(a) Note that $C(q,r) = r \cdot z(r,q)$, thus $\frac{\partial C}{\partial r_j} = z_j$.

$$\frac{\partial MC}{\partial r_j} = \frac{\partial}{\partial r_j}\frac{\partial C}{\partial q} = \frac{\partial}{\partial q}\frac{\partial C}{\partial r_j} = \frac{\partial z_j}{\partial q}$$

This quantity is greater than 0 if and only if z_j is normal input, which means that the marginal cost will increase with an increase in r_j if and only if z_j is normal.

(b) Note that

$$\frac{\partial}{\partial r_j}AC(q) = \frac{\partial}{\partial r_j}\left(\frac{C}{q}\right) = \frac{1}{q}\frac{\partial C}{\partial r_j} = \frac{z_j}{q}$$

Using this together with the result of (a), we have

$$\frac{\partial MC(q)}{\partial r_j}\Bigg/\frac{\partial AC(q)}{\partial r_j} = \frac{\partial z_j}{\partial q}\Bigg/\frac{z_j}{q} = E(z_j,q)$$

(c) With free entry, every firm produces at the minimum of AC curve, where $MC(q) = AC(q)$ always holds true. This is an identity of r. Differentiating this with respect to r_j,

$$\begin{aligned}
\frac{\partial MC(q)}{\partial r_j} + \frac{\partial MC(q)}{\partial q}\frac{dq}{dr_j} &= \frac{\partial AC(q)}{\partial r_j} + \frac{\partial AC(q)}{\partial q}\frac{dq}{dr_j} \\
\frac{\partial^2 C(q)}{\partial q^2}\frac{dq}{dr_j} &= \frac{\partial AC(q)}{\partial r_j} - \frac{\partial MC(q)}{\partial r_j} \qquad \left(\because \frac{\partial AC(q)}{\partial q} = 0\right) \\
\frac{dq}{dr_j} &= \left(\frac{\partial AC(q)}{\partial r_j} - \frac{\partial MC(q)}{\partial r_j}\right)\Bigg/\frac{\partial^2 C(q)}{\partial q^2}
\end{aligned}$$

Since C is convex in q, this is positive if and only if $\frac{\partial AC(q)}{\partial r_j} \geq \frac{\partial MC(q)}{\partial r_j}$, which is equivalent to $E(z_j,q) \leq 1$. A trivial case is that z_j is an inferior input. So if $E(z_j,q) \leq 1$, the output of each active firm, q, rises as the result of increase in r_j.

(d) To produce $q = F(z)$ units of output, the optimal input vector satisfies

$$\frac{\alpha}{\beta}\cdot\frac{z_2}{z_1-\gamma} = \frac{r_1}{r_2} \quad \text{and} \quad q = (z_1-\gamma)^{\alpha}z_2^{\beta}$$

so

$$z_1 = \gamma + q^{\frac{1}{\alpha+\beta}}\left(\frac{\alpha}{\beta}\cdot\frac{r_2}{r_1}\right)^{\frac{\beta}{\alpha+\beta}} \quad \text{and} \quad z_2 = q^{\frac{1}{\alpha+\beta}}\left(\frac{\beta}{\alpha}\cdot\frac{r_1}{r_2}\right)^{\frac{\alpha}{\alpha+\beta}}$$

From

$$\ln z_2 = \frac{1}{\alpha+\beta}\ln q + const$$

we have

$$E(z_2,q) = \frac{\partial \ln z_2}{\partial \ln q} = \frac{1}{\alpha+\beta} > 1$$

$E(z_1,q) < 1$ since $k_1E(z_1,q) + k_2E(z_2,q) = 1$[1] where k_i is the cost share of input i that satisfies $k_1 + k_2 = 1$. Therefore, from the result of (c), when r_1 increases, the output of an active firm rises.

[1] This relation holds only when the technology shows constant returns to scale. Note that it is locally CRS when q is an AC minimizing output.

2003 Fall 2. The big Game

(a) Let $v(c) = \ln c$. The measure of relative risk aversion is

$$R(c) = -\frac{v''(c)c}{v'(c)} = -\frac{-c^{-2} \cdot c}{c^{-1}} = 1$$

So $v(c) = \ln c$ is indeed a VNM utility function for each fan.

(b) Let $W^B = W^T = W$. Total wealth is $2W$ in any state. But utility function is not homothetic. Let state 1 be the case when the Bruins win, and state 2 be the other case. Bruin fans have a utility function $u(c_1^B, c_2^B) = 0.8 \ln c_1^B + 0.2 \ln c_2^B$. Trojan fans have $u(c_1^T, c_2^T) = 0.4 \ln c_1^T + 0.6 \ln c_2^T$. All fans are maximizing their utility under the budget constraint.[2]

$$\max_{c_1, c_2} u(c_1, c_2) \qquad \text{subject to } p_1 c_1 + p_2 c_2 = W$$

FOC's are

$$\frac{0.8}{c_1^B} = \frac{0.4}{c_1^T} = \lambda p_1 \quad \text{and} \quad \frac{0.2}{c_2^B} = \frac{0.6}{c_2^T} = \lambda p_2$$

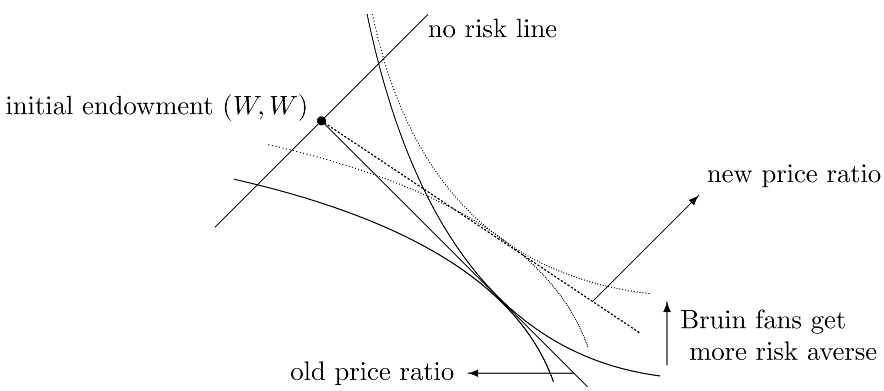
Using these together with budget constraints as well as market clearing conditions ($c_1^B + c_1^T = c_2^B + c_2^T = 2W$), we have

$$c_1^B = \frac{4}{3}W \ , \ c_2^B = \frac{1}{2}W \ , \ c_1^T = \frac{2}{3}W \ , \ c_2^T = \frac{3}{2}W \ , \ \frac{p_1}{p_2} = \frac{3}{2}$$

So the first statement is TRUE. But Bruin fans bet $\frac{1}{2}W$ which is a half of their wealth, and Trojan fans bet $\frac{1}{3}W$ which is a third of their wealth, so the second statement is FALSE.

(c) Suppose there are Bruin fans only. (Total wealth of Trojan fans is close to 0 relatively.) Then there is no aggregate risk, and utility function is identically $u(c_1, c_2) = 0.8 \ln c_1 + 0.2 \ln c_2$. So the equilibrium odds will be 4:1 in favor of the Bruins. Therefore, if Bruin fans have more total wealth than Trojan fans, the equilibrium odds will be between 3:2 and 4:1. (For the details, see Jiyeon) This seems reasonable. If the number of Bruin fans increases, more people will bet on the Bruins, a ticket for the Bruins gets more expensive, so the equilibrium odds will be higher in favor of the Bruins.

(d) The market odds will go down. Intuitively, when Bruin fans are more risk averse, they are reluctant to be far from no risk line, which means that they will bet less on the Bruins. This cuts off the price of a ticket for the Bruins, so the equilibrium odds will be less than before.



[2]Note that here we implicitly normalize prices such that $p_1 + p_2 = 1$. To see this, consider the case in which they don't bet on any case, which is feasible. In that case, they choose $c_1 = c_2 = W$, so $p_1 + p_2 = 1$ should be satisfied for the budget constraint to hold.