2005 Fall 5. Ordinal versus Cardinal Interpretations of Efficiency

(a) omitted. (It looks obvious, but a diagram doesn't help understand)

(b) It would be okay just to check that under those prices, markets clear. $L = 1$, and $K = 2$, so

$$\begin{aligned} p_K &= MP_K|_{L=1,K=2} = 2^{\frac{1}{2}}1^{\frac{1}{2}}2^{-\frac{1}{2}} = 1 \\ w &= MP_L|_{L=1,K=2} = 2^{\frac{1}{2}}1^{-\frac{1}{2}}2^{\frac{1}{2}} = 2 \\ \text{output} &= 2^{\frac{3}{2}}1^{\frac{1}{2}}2^{\frac{1}{2}} = 4 \end{aligned}$$

Employed people has an income of $p_K + w = 3$, so consumes 3 units of commodity, and unemployed peopel has an income of $p_K = 1$, so consumes 1 unit of commodity. Therefore consumption market clears. Note that (1,0) and (3,1) are indifferent since $\Delta(c) = 1 + c$.

(c) I assumed that the question means linear preferences. Clearly, the allocation and prices in (b) still satisfy market clearing condition. It is okay if we check that we cannot make anybody better off by choosing another allocation. If one chooses $L \in (0, 1)$, he would get an income of $p_K + wL = 1 + 2L$, so consumes $1 + 2L$ units of commodity. Here linear preferences imply that $(1+2L, L) = (1-L)(1, 0)+L(3, 1)$ is indifferent to (1,0) and (3,1). So he cannot be better off.

Now we can find another equilibrium allocation, for example, that everybody chooses $L = 0.5$, and consumes 2 units of commodity. Markets clear with these allocation, too, since the aggregate labor and output don't change.

(d) Let $u(c', 1) = u(c, 0)$, then $\Delta(c) = c' - c$ by definition.

$$\log(1 + c') - \log 2 = \log(1 + c)$$

$$\log(1 + c') = \log(2 + 2c)$$

$$c' - c = 1 + c$$

as desired.

(e) The allocation in (b) is Pareto optimal among feasible points. But if we introduce a lottery of having to work with probability $\frac{1}{2}$, it would be another possibility that we couldn't choose before. So it's no surprise if we can be better off now.

2005 Fall 6. Public versus Private Goods

(a) For PUB, it would be optimal to choose $s = 1$ if and only if $\sum_{i=1}^{n} v_i \geq 0$. (In case of equality, it could be $s = 0$.)

For PRI, $s = s_i$ for all $j \neq i$ when $v_i = \max_j\{v_j\}$. In other words, the agent with the highest valuation gets the commodity.

(b) For agent i to reveal their true v_i,

$$v_i(s(v)) - p_i(v) \geq v_i(s(v_i', v_{-i})) - p_i(v_i', v_{-i}) \quad \forall v_i',\ \forall i$$

has to hold. In other words, he has to get bigger utility when he truthfully reports. These conditions are the same for both PUB and PRI.

(c) We can use VCG mechanism. It guarantees incentive compatibility. Let $-p_i(v) = \sum_{j \neq i} v_j(s(v)) - h(v_{-i})$ where h is an arbitrary function. Then

$$v_i(s(v)) - p_i(v) = \sum_{j=1}^{n} v_j(s(v)) - h(v_{-i}) = g(v) - h(v_{-i})$$

and by definition of g, he would report v_i, true valuation.[1] We can use $h(v_{-i}) = g(v_{-i})$, which is MP mechanism. These arguments are the same for both PUB and PRI.

(d) With MP mechanism, in PUB case, $-p_i(v) \leq 0$ by definition of $g(v_{-i})$. So

$$\sum_{i=1}^{n} p_i(v) \geq 0$$

satisfies weak budget balance. But $v_i(s(v)) - p_i(v)$ could be negative when v_i is negative, so the machanism is not valuntary. Note that PRI case is an auction problem.

Here MP mechanism is English auction. In English auction, everybody gets nonnegative payoff, so the mechanism is voluntary. The mechanism satisfies weak budget balance, too, since $-p_i(v) \leq 0$ in English auction. Note that we are not considering the seller here, which is different from the double auction model.

(e) As $n \to \infty$,

$$\frac{1}{n} \sum_{i=1}^{n} p_i(v) \to 0$$

in both cases. This means that nobody can affect the outcome as the number of individuals increases.

[1] By definition of $s(v)$, for any feasible s',

$$\sum_{j=1}^{n} v_j(s(v)) \geq \sum_{j=1}^{n} v_j(s')$$

Take $s' = s(v_i', v_{-i})$, then

$$\sum_{j=1}^{n} v_j(s(v)) \geq \sum_{j=1}^{n} v_j(s(v_i', v_{-i}))$$

which means that reporting true v_i maximizes $g(v)$.