2005 Spring 1. Short-takes

(a) First Law of Supply means that when a price of output rises, a supply of that commodity increases. This can be proven in two cases. One is the case in which the firm is a monopolist, and the other is when firms are price takers. Here we prove the case of price taking firms. Let z^i and q^i be the input and output vectors when the input and output prices are r^i and p^i. Note that z^i is feasible to produce q^i. Then profit maximization hypothesis implies that

$$p^0 \cdot q^0 - r^0 \cdot z^0 \geq p^0 \cdot q^1 - r^0 \cdot z^1$$
$$p^1 \cdot q^1 - r^1 \cdot z^1 \geq p^1 \cdot q^0 - r^1 \cdot z^0$$

Adding up term by term and organizing,

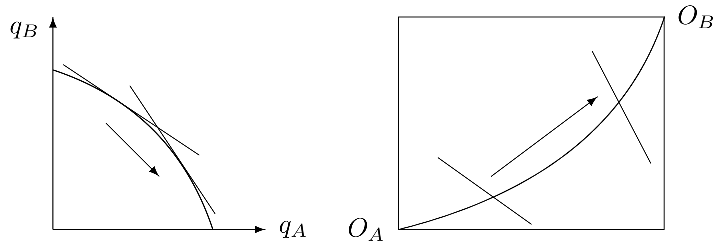
$$(p^0 - p^1) \cdot (q^0 - q^1) - (r^0 - r^1) \cdot (z^0 - z^1) \geq 0$$

Suppose only p changes from p^0 to p^1 ($p^0 < p^1$) and r doesn't change. From above inequality,

$$(p^0 - p^1) \cdot (q^0 - q^1) \geq 0$$

So $q^0 - q^1 \leq 0$, or $q^0 \leq q^1$, which completes the proof.

(b) That commodity A is more input 1 intensive means that MRTS increases as output of A rises along the production efficient points. Since production possibility set is (strictly) convex when the production function is quasi concave and constant returns to scale, a change in preferences in favor of commodity A raises the relative price of A and output of A. Increase in output of A means that MRTS should increase, which is equilvalent to that the relative price of input 1 increases.



(c) Consider a homothetic and identical preferences. We can introduce a representative agent. The discount factor is δ. Suppose that there is no storage. Then, the max problem is

$$\max_{c_1,c_2} u(c_1) + \delta u(c_2) \qquad \text{subject to } p \cdot c \leq p \cdot \omega$$

From FOC's, we have the following equilibrium condition

$$\delta^{t-1} \frac{u'(\omega_{tj})}{p_{tj}} = \lambda \quad \text{(constant)} \qquad \text{for all } t, j$$

Now suppose that storage is possible. If $p_{2j} > p_{1j}$, people will store some of commodity j. Storage will stop at the point at which $p_{2j} = p_{1j}$. If $p_{2j} < p_{1j}$, people will not store any of commodity j, in which case, the equilibrium spot and futures prices of commodity j are not the same. We need to find the condition under which no storage equilibrium price satisfies $p_{2j} \geq p_{1j}$. This is equivalent to

$$\delta \frac{u'(\omega_{2j})}{\lambda} \geq \frac{u'(\omega_{1j})}{\lambda}$$

or

$$\delta u'(\omega_{2j}) \geq u'(\omega_{1j})$$

2005 Spring 2. Individual and aggregate risk

(a) Fix $W_1, \cdots, W_S$. Note that utility function is Cobb-Douglas. For given W_s, utility maximizing consumption is

$$c_{1s} = \frac{\alpha W_s}{p_{1s}} \text{ and } c_{2s} = \frac{(1-\alpha)W_s}{p_{2s}}$$

So from W_s, he can get utility

$$v(c_{1s}, c_{2s}) = \left(\frac{\alpha W_s}{p_{1s}}\right)^{\alpha} \left(\frac{(1-\alpha)W_s}{p_{2s}}\right)^{1-\alpha}$$

So indirect utility is

$$U(W_1, \cdots, W_S) = \sum_{s=1}^{S} \pi_s \left(\frac{\alpha W_s}{p_{1s}}\right)^{\alpha} \left(\frac{(1-\alpha)W_s}{p_{2s}}\right)^{1-\alpha}$$

(b) Now we allocate expenditure amount on each state. Utility function can be written as follows.

$$U(W_1, \cdots, W_S) = \sum_{s=1}^{S} \pi_s \left(\frac{\alpha}{p_{1s}}\right)^{\alpha} \left(\frac{1-\alpha}{p_{2s}}\right)^{1-\alpha} W_s$$

Note that this is a linear function in W_s. So he will buy only claims to state 1 if

$$\pi_1 \left(\frac{\alpha}{p_{11}}\right)^{\alpha} \left(\frac{1-\alpha}{p_{21}}\right)^{1-\alpha} \geq \pi_s \left(\frac{\alpha}{p_{1s}}\right)^{\alpha} \left(\frac{1-\alpha}{p_{2s}}\right)^{1-\alpha} \qquad \text{for all } s$$

or

$$\frac{\pi_1}{\pi_s} \geq \left(\frac{p_{11}}{p_{1s}}\right)^{\alpha} \left(\frac{p_{21}}{p_{2s}}\right)^{1-\alpha} \qquad \text{for all } s$$

(c) Note that utilty is homothetic and identical, and that all consumers have identical beliefs. All consumers will consume a fraction of the aggregate endowment. If the aggregate endowment in state s is $\omega_s = (\theta_s \beta_1, \theta_s \beta_2)$, then the representative agent's utility is

$$u(\omega_1, \cdots, \omega_S) = \sum_{s=1}^{S} \pi_s \left(\theta_s \beta_1\right)^{\alpha} \left(\theta_s \beta_2\right)^{1-\alpha} = \sum_{s=1}^{S} \pi_s \theta_s \beta_1^{\alpha} \beta_2^{1-\alpha} = \beta_1^{\alpha} \beta_2^{1-\alpha}$$

If there is no risk and the aggregate endowment is $\omega = (\beta_1, \beta_2)$, then the representative agent's utility is

$$u(\omega) = \beta_1^{\alpha} \beta_2^{1-\alpha}$$

Utilities are the same in both cases, so for all comsumers they will be the same too.

(d) If the aggregate endowment in state s is $\omega_s = (\beta_1, \theta_s \beta_2)$, then the representative agent's utility is

$$u(\omega_1, \cdots, \omega_S) = \sum_{s=1}^{S} \pi_s \beta_1^{\alpha} \left(\theta_s \beta_2\right)^{1-\alpha} = \beta_1^{\alpha} \beta_2^{1-\alpha} \sum_{s=1}^{S} \pi_s \theta_s^{1-\alpha}$$

This is less than $u(\omega)$, the utility in no risk case. To see this, note that $f(x) = x^{1-\alpha}$ is concave for any $\alpha > 0$ and that $\sum_{s=1}^{S} \pi_s = 1$, so

$$\sum_{s=1}^{S} \pi_s \theta_s^{1-\alpha} = \sum_{s=1}^{S} \pi_s f(\theta_s) \leq f\left(\sum_{s=1}^{S} \pi_s \theta_s\right) = f(1) = 1$$

Equilibrium prices will be

$$p_{1s} = \left.\frac{\partial u}{\partial c_{1s}}\right|_{\omega} = \pi_s \alpha \left(\frac{\theta_s \beta_2}{\beta_1}\right)^{1-\alpha} \quad \text{and} \quad p_{2s} = \left.\frac{\partial u}{\partial c_{2s}}\right|_{\omega} = \pi_s (1-\alpha) \theta_s \left(\frac{\beta_1}{\theta_s \beta_2}\right)^{\alpha}$$

(Question: characterize equilibrium prices "again"?)