

## 2006 Fall 1. Walrasian Equilibrium and Time

(a) True.

In both cases, there are  $N \times T$  markets in the economy, so we can optimize if there are  $N \times T - 1$  relative prices.

(b) True.

We need  $N \times T - 1$  relative prices to optimize. If  $N$  markets are open each period and  $T - 1$  bonds markets of all possible maturity dates also are open, then we have  $(N - 1) \times T + (T - 1) = N \times T - 1$  relative prices, so we have no problem.<sup>1</sup>

(c) False.

For infinite horizon problem, we need to consider some conditions under which the max problem holds, for example, bounded utility function and transversality conditions.

(d) True.

Different beliefs are incorporated into consumers' utility functions, so there is no problem to set up the model and define an equilibrium. But it is not easy to find such an equilibrium, since the utility function is no more identical, so we cannot introduce a representative agent.

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<sup>1</sup>We always need, however, the assumption that the distribution of endowment in each period (and state) should be perfectly anticipated. If this fails, then even when there exist complete futures markets, we cannot optimize consumption ex post, in the sense that we would have to reoptimize by trading in the future spot markets.

**2006 Fall 2. Monopoly response**

(a)

(i) True

From the first law of supply and input demand, the demand for input 1 falls if the price of input 1 rises. Let  $z^0$  be the profit maximizing input vector under  $r^0$ , and  $z^1$  be that under  $r^1$ . Let also  $R(\cdot)$  the revenue function. From the profit maximizing condition,

$$\begin{aligned} R(z^0) - r^0 \cdot z^0 &\geq R(z^1) - r^0 \cdot z^1 \\ R(z^1) - r^1 \cdot z^1 &\geq R(z^0) - r^1 \cdot z^0 \end{aligned}$$

Summing two inequalities side by side, we have

$$\begin{aligned} R(z^0) + R(z^1) - r^0 \cdot z^0 - r^1 \cdot z^1 &\geq R(z^0) + R(z^1) - r^0 \cdot z^1 - r^1 \cdot z^0 \\ (r^0 - r^1) \cdot (z^0 - z^1) &\leq 0 \end{aligned}$$

Thus if  $r_1$  rises from  $r_1^0$  to  $r_1^1$  and others are equal,

$$(r_1^0 - r_1^1) \cdot (z_1^0 - z_1^1) \leq 0$$

Thus  $z_1^0 \leq z_1^1$ .

(ii) True

This is called ‘‘Le Chatelier Principle.’’ The first thing to do is to show that profit function  $\pi(r) = R[z(r)] - r \cdot z(r)$  is convex. Let  $z^\lambda = (1 - \lambda)z^0 + \lambda z^1$  be the profit maximizing input vector under  $r^\lambda = (1 - \lambda)r^0 + \lambda r^1$ . In similar way in (a),

$$\begin{aligned} \pi(r^0) &= R(z^0) - r^0 \cdot z^0 \geq R(z^\lambda) - r^0 \cdot z^\lambda \\ \pi(r^1) &= R(z^1) - r^1 \cdot z^1 \geq R(z^\lambda) - r^1 \cdot z^\lambda \end{aligned}$$

Summing two inequalities with weights  $1 - \lambda$  and  $\lambda$  respectively, we have

$$(1 - \lambda)\pi(r^0) + \lambda\pi(r^1) \geq R(z^\lambda) - [(1 - \lambda)r^0 + \lambda r^1] \cdot z^\lambda$$

Since the right hand side is equal to  $\pi(r^\lambda)$ ,  $\pi$  is convex. By Envelope theorem, we have

$$\frac{\partial \pi}{\partial r_j}(r) = -z_j \tag{1}$$

Denote as  $\pi^L$  the profit function in the long run, and as  $\pi^S$  that in the short run, then

$$\pi^L(r) \geq \pi^S(r)$$

since we have more options in the long run than in the short run. Also,

$$\pi^L(r^0) = \pi^S(r^0)$$

thus we have

$$\frac{\partial^2 \pi^L}{\partial r_1^2}(r^0) \geq \frac{\partial^2 \pi^S}{\partial r_1^2}(r^0)$$

From (1),

$$-\frac{\partial z_1^L}{\partial r_1}(r^0) \geq -\frac{\partial z_1^S}{\partial r_1}(r^0)$$

that is, the long run demand will change more than the short run demand when the input price changes. Notice that  $-\frac{\partial z_1}{\partial r_1}(r) \geq 0$ .

(iii) False.

The firms output may increase in some cases. To see this, note that  $C(q, r) = r \cdot z(r, q)$ , thus

$$\frac{\partial MC}{\partial r_1} = \frac{\partial^2 C}{\partial r_1 \partial q} = \frac{\partial}{\partial q} \frac{\partial C}{\partial r_1} = \frac{\partial z_1}{\partial q}$$

This quantity is greater than 0 only if  $z_1$  is normal input. If  $z_1$  is inferior input, this can be negative, in which case increase of  $r_1$  shifts down MC curve, in turn increasing optimal  $q$ .

(iv) True.

This depends on two factors. One is substitutability between inputs, and the other is whether each input is normal or inferior. Suppose all inputs are non-substitutable. (Consider a Leontief production function.) Then, all inputs are normal. If  $r_1$  increases, optimal output decreases, so demands for all inputs decrease. Suppose  $z_1$  is inferior and all inputs are highly substitutable. If  $r_1$  increases, optimal output increases, so demands for all inputs increase. High substitutability also raises demands for other inputs.

(b)

The results do not change.

(i) Let  $z^0$  be the profit maximizing input vector under  $r_1^0$ , and  $z^1$  be that under  $r_1^1$ . If the firm is also a price setter in the other input markets, from the profit maximization condition,

$$\begin{aligned} R(z^0) - [r_1^0 z_1^0 + r_2(z_2^0)z_2^0 + \cdots + r_n(z_n^0)z_n^0] &\geq R(z^1) - [r_1^0 z_1^1 + r_2(z_2^1)z_2^1 + \cdots + r_n(z_n^1)z_n^1] \\ R(z^1) - [r_1^1 z_1^1 + r_2(z_2^1)z_2^1 + \cdots + r_n(z_n^1)z_n^1] &\geq R(z^0) - [r_1^1 z_1^0 + r_2(z_2^0)z_2^0 + \cdots + r_n(z_n^0)z_n^0] \end{aligned}$$

Summing two inequalities side by side, we have

$$(r_1^0 - r_1^1) \cdot (z_1^0 - z_1^1) \leq 0$$

If  $r_1^0 < r_1^1$ , then  $z_1^0 \geq z_1^1$ .

(ii) Convexity of  $\pi$ , Envelope theorem and  $\pi^L \geq \pi^S$  also hold here, thus nothing changes. Therefore a demand for input 1 will change more in the long run than in the short run under this assumption as well. However there is nothing to say about the effect of the change of  $r_1$  on demand for the other inputs.