2006 Fall 5. Incentive-Efficiency Tradeoffs in Teams

(a) For efficiency, maximize the total surplus

$$\max 3(e_1 e_2)^{\frac{1}{3}} - \sigma_1 e_1 - \sigma_2 e_2$$

FOC's are

$$e_1^{-\frac{2}{3}} e_2^{\frac{1}{3}} = \sigma_1 \quad , \quad e_1^{\frac{1}{3}} e_2^{-\frac{2}{3}} = \sigma_2$$

Solving these equations,

$$e_1 = \sigma_1^{-2} \sigma_2^{-1} \quad , \quad e_2 = \sigma_1^{-1} \sigma_2^{-2}$$

(b) FOC's of individual optimization are

$$\frac{\partial y_1}{\partial e_1} = \sigma_1 \quad , \quad \frac{\partial y_2}{\partial e_2} = \sigma_2$$

but FOC of gross optimization is

$$\frac{\partial y_1}{\partial e_1} + \frac{\partial y_2}{\partial e_1} = \sigma_1 \quad , \quad \frac{\partial y_1}{\partial e_2} + \frac{\partial y_2}{\partial e_2} = \sigma_2$$

So in general, they don't coincide with each other. If we use a fraction rule, for example,

$$y_1 = \alpha y \quad , \quad y_2 = (1 - \alpha) y$$

then it is easy to prove that FOC's don't coincide.

(c) Use MP mechanism so that

$$y_1 = y(e_1, e_2) - \frac{1}{2} y(\bar{e}_1, e_2) \quad , \quad y_2 = y(e_1, e_2) - \frac{1}{2} y(e_1, \bar{e}_2)$$

Then clearly, both agents would choose $\bar{e}_1$ and $\bar{e}_2$ maximizing $y(e_1, e_2)$. This scheme has a problem that we have to know $\bar{e}_1$ and $\bar{e}_2$ exactly. Also for some level of e_1 and e_2 (higher than $\bar{e}_1$ and $\bar{e}_2$), even one-sided balance may not hold.

(d) No. The above scheme can be used only if we can calculate $\bar{e}_1$ and $\bar{e}_2$. If we don't know σ_1 and σ_2, we cannot calculate them. Of course there are such reward schemes if we don't have to satisfy budget balance. For example,

$$y_1 = y_2 = y(e_1, e_2)$$

This would make every agent reveal their true σ. For onde-sided balance, there still could be such a mechanism. Assume that there are upper bounds for the level of effort, say $\widehat{e}_1$ and $\widehat{e}_2$. Then we can use

$$y_1 = y(e_1, e_2) - \frac{1}{2} y(\widehat{e}_1, e_2) \quad , \quad y_2 = y(e_1, e_2) - \frac{1}{2} y(e_1, \widehat{e}_2)$$

This scheme satisfies

$$y_1(e_1, e_2) + y_2(e_1, e_2) \leq y(e_1, e_2)$$

for all e_1 and e_2 as long as y is an increasing function. Under this scheme, every agent reveals their true σ. But here individual rationality may not hold for some e_1 and e_2.

2006 Fall 6. Cournot Monopolistic Competition with Large Numbers

(a) When $N = 1$ and $B_1 = C_1 = 1$, then, agent 2's demand for commodity 1 is

$$10 - z_{21} = p_1$$

If agent 1 doesn't limit capacity constraint, his marginal cost would be $MC_1 = z_1$, so a PTE output is 5 and profit is

$$\pi_1 = 5 \cdot 5 - \frac{1}{2}5^2 = \frac{25}{2}$$

But he can optimally limit capacity. Note that the marginal revenue would be

$$MR_1 = 10 - 2z_{21}$$

If he choose $z = \frac{10}{3}$ from $MR_1 = MC_1$ as a capacity constraint, the price would be $\frac{20}{3}$, so

$$\pi_1^K = \frac{20}{3} \cdot \frac{10}{3} - \frac{1}{2}\left(\frac{10}{3}\right)^2 = \frac{150}{9}$$

The same thing is true for agent 2.

(b) When $B_N = 1$, an individual demand and MR would be the same with one in (a).

$$z_{j1} = 10 - p_1$$

The market demand for commodity 1 will be

$$Z = \sum_{j \neq 1} z_{j1} = N(10 - p_1)$$

This becomes horizontal line as N goes to infinity. MR would be

$$MR_1 = 10 - \frac{2}{N}Z$$

and this also becomes horizontal line. So in (i) and (iv), agents would not want to impose capacity constraint as N goes to infinity.

When $B_N = N$, and individual demand is

$$Nz_{j1} = 10 - p_1$$

and thus the market demand is

$$Z = 10 - p_1$$

which doesn't become horizontal as N goes to infinity. So in (ii) and (iii), agents would want to impose capacity constraint as in (a).