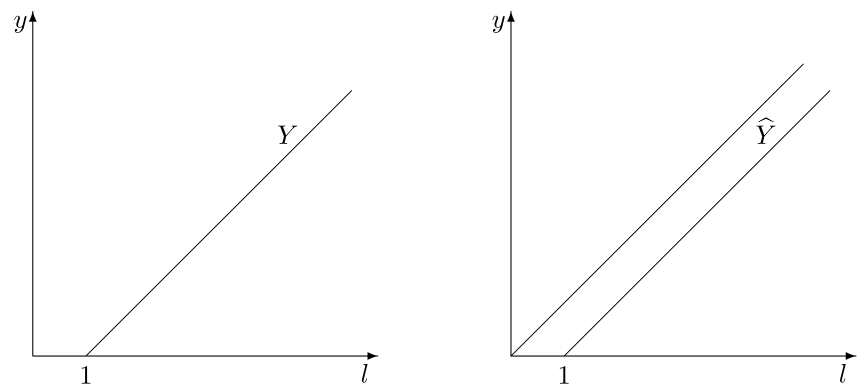
2006 Spring 5. Public versus Private Goods

See Hong's TA note.
See Comp 2005 Fall 6.

2006 Spring 6. Inter-regional Trade

I doubt this is a question we learned in the class, but one question on the final was similar.

(a)



Note that $\widehat{Y}$ includes $y = l$ in it. There is no such scale at which Y coincides with $\widehat{Y}$. If we scale Y by any scalar k, it does not coincide with $\widehat{Y}$.

(b) If the technology is $\widehat{Y}$, then we can choose $y = l$. Since every region has the same CRS technology, there is no need to trade between regions.[1]

(c) Since the technology is now IRS,[2] it would be efficient to produce whole output in one region. With two regions, let each region produce one variety of J commodities with 2 units of labor for each commodity. Each region can consume 2 varieties of each of J commodities. Each variety of commodity would be produced at the level of 1, so each region can consume $\frac{1}{2}$ unit of each commodity.[3]

$$g_2 \geq \sum_{j=1}^{J} \left[\sum_{k=1}^{2} \left(\frac{1}{2} \right)^{\beta} \right]^{\frac{1}{2}} = 2^{\frac{1}{2}(1-\beta)} J$$

If there is only one region, the region would want to maximize utility by equally producing each type of commodities. Actually the maximum utility is obtained by producing 1 unit of only one variety of each commodity, I guess. For example, let's take an example $J = 2$ and $K = 2$. The region can produce 1 unit of only one variety of each commodity, which yields $u = 2$. Or it can produce 0.5 unit of commodity 1, and 0.25 unit of variety 1 of commodity 2, and 0.25 unit of variety 2 of commodity 2. Here utility is $u = 0.5^{0.5\beta} + 2^{0.5} 0.25^{0.5\beta}$ which gives less than 2 even when $\beta = \frac{1}{2}$. I cannot prove this formally.

$$g_1 = \sum_{j=1}^{J} \left(1^{\beta} \right) = J$$

[1] CRS technology means constant MC and AC, which enables anybody to produce efficiently.

[2] The first 1 unit of labor produces nothing, but then every unit after that produces 1.

[3] There could be another production plan. For example, each region produces 3 units of each variety of $\frac{J}{2}$ commodities with 4 units of labor, which enables each region to consume 1.5 unit of each variety of all commodities. This yields utility of $1.5^{0.5\beta} J$. This might be or might not be larger than the one we get in the main text.

Obviously, $g_2 > g_1$.

(d) Here I assumed $N = K$. Since there are enough regions, they can produce every commodity, and consume equally. Note that there are $2NJ$ labors. Each variety of each commodity would be produced at the level of 1, and each region consumes $\frac{1}{R} = \frac{1}{NJ}$ unit of the commodity.

$$g_{NJ} = \sum_{j=1}^{J} \left[\sum_{k=1}^{N} \left(\frac{1}{NJ} \right)^{\beta} \right]^{\frac{1}{2}} = N^{\frac{1}{2}(1-\beta)} J^{1-\frac{1}{2}\beta}$$

Let $N \to \infty$, then

$$\begin{aligned} \lim_{N\to\infty} \left[g_{NJ} - g_{(N-1)J} \right] &= \lim_{N\to\infty} J^{1-\frac{1}{2}\beta} \left[N^{\frac{1}{2}(1-\beta)} - (N-1)^{\frac{1}{2}(1-\beta)} \right] \\ &\leq J^{1-\frac{1}{2}\beta} \lim_{N\to\infty} \frac{1}{2}(1-\beta)(N-1)^{\frac{1}{2}(1-\beta)-1}[N-(N-1)] \\ &= 0 \end{aligned}$$

The second inequality comes from the fact that $f(x) = x^{\frac{1}{2}(1-\beta)}$ is concave. Note that for concave function f, $f(x^1) - f(x^0) \leq f'(x^0)(x^1 - x^0)$. Since $g_{NJ} - g_{(N-1)J} \geq 0$,

$$\lim_{N\to\infty} \left[g_{NJ} - g_{(N-1)J} \right] \geq 0$$

and thus

$$\lim_{N\to\infty} \left[g_{NJ} - g_{(N-1)J} \right] = 0$$

As $N \to \infty$, the above allocation with trade does not converge to one in (b), since here 2 units of labor is used to produce 1 unit for all varieties of all commodities, whereas in (b), 1 unit of labor is necessary to produce 1 unit in all cases.[4]

[4] This is my guess.