Rules for price discrimination

\[ MR_1 = MR_2 = MC \]

If there is a restriction (e.g. arbitrage, regulation), set up a constrained maximization problem.

How to find a constraint? Think reasonably. (See the solution for the original problem without any restriction.)

\[
\text{max } P_1 Q_1 + P_2 Q_2 - C(Q_1 + Q_2) \quad \text{subject to } P_2 = P_1 - 5
\]

\[ \mathcal{L} = P_1 Q_1 + P_2 Q_2 - C(Q_1 + Q_2) + \lambda (P_2 - P_1 + 5) \]

FOC with respect to variables?

Simplify further by plugging \( Q_1 = 55 - P_1, Q_2 = 70 - 2P_2 \) and \( P_2 = P_1 - 5 \) into the objective function.

\[
\text{max } P_1 (55 - P_1) + P_2 (70 - 2P_2) - C(55 - P_1 + 70 - 2P_2)
\]

Exercise: Question 14.6 in the textbook.

Rules for two part tariff

\( P = MC \) for usage fee

\( T = \text{consumer surplus (CS)} \) for entry fee

If there are two types of consumers with different demand function, consider

1. \( P = MC, T = \text{small CS} \) : both types pay \( T \).
2. \( P = MC, T = \text{big CS} \) : only higher type pays \( T \).
3. \( P > MC \) and \( T = \text{small CS} \) is a function of \( P \).

Find \( P \) that maximizes total profit.

⇒ Find which of the above three gives the maximum profit.

Useful tip: Usually (1) does not maximize profit.

Exercise: Question solved in the TA session.

Answer for 14.6

(a) \( Q_1 = 25 \) and \( P_1 = 30 \)

\( Q_2 = 30 \) and \( P_2 = 20 \)

profit = 1075

(b) \( Q_1 = \frac{45}{3} \) and \( P_1 = \frac{80}{3} \)

\( Q_2 = \frac{80}{3} \) and \( P_2 = \frac{65}{3} \)

(\( \lambda = \frac{20}{3} \) or \( -\frac{20}{3} \) if Lagrangian method used)

profit = \( \frac{3150}{3} \approx 1050 \)

(c) \( P_1 = P_2 = \frac{70}{3} \)

\( Q_1 = \frac{25}{3} \) and \( Q_2 = \frac{70}{3} \)

profit = \( \frac{3095}{3} \approx 1005 \)

(d) \( P = 5 \) in both markets.

\( T_1 = 1250 \) and \( T_2 = 900 \)

profit = 2150