TA supplementary note 2
Yang, April 16th.

Rules for mixed strategy Nash Equilibrium

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,1</td>
<td>0,4</td>
</tr>
<tr>
<td>B</td>
<td>0,1</td>
<td>3,0</td>
</tr>
</tbody>
</table>

Let it be mixed strategy Nash Equilibrium that player 1 plays A with probability \(a\), and B with probability \(1-a\), and player 2 plays C with probability \(c\), and D with probability \(1-c\).

1st method: Make the other player indifferent between two actions. (simple but not sufficient)

\[ U_1(A) = c \cdot 2 + (1-c) \cdot 0 = 2c \quad \text{because when 1 plays A, 1 gets 2 with prob } c, \text{ and 0 with } 1-c. \]
\[ U_1(B) = c \cdot 0 + (1-c) \cdot 3 = 3-3c \]

Under strictly mixed strategy Nash Equilibrium, player 2 wants to make player 1 indifferent between A and B, since otherwise player 1 will choose either A or B with probability 1, without randomizing over A and B. This yields \(2c = 3 - 3c\), or \(c = \frac{3}{5}\). For the same reason,

\[ U_2(C) = a \cdot 1 + (1-a) \cdot 1 = 1 \quad \text{because when 2 plays C, 2 gets 1 with prob } a, \text{ and 1 with } 1-a. \]
\[ U_2(D) = a \cdot 4 + (1-a) \cdot 0 = 4a \]

and \(U_2(C) = U_2(D)\) yields \(a = \frac{1}{4}\).

2nd method: Find BEST RESPONSE functions of both players. (complex but complete)

Follow the instruction in the lecture note, which is explained in pages 25-28. More examples are given in pages 29-33.

Answer for 8.1
(a) (C,F)
(b) Player 1 plays A with probability \(\frac{1}{2}\), and B with probability \(\frac{1}{2}\). Player 2 also plays D with probability \(\frac{1}{2}\), and E with probability \(\frac{1}{2}\).
(c) Under (C,F), both players get 4. Under the strictly mixed strategy Nash Equilibrium in (b), player 1 gets 6 in expectation, and player 2 gets 7 in expectation.
(d) omitted.

Answer for 8.2
Wife plays Ballet with probability \(\frac{K}{K+1}\), and Boxing with probability \(\frac{1}{K+1}\). Husband plays Ballet with probability \(\frac{1}{K+1}\), and Boxing with probability \(\frac{K}{K+1}\)