Cournot Competition among heterogeneous firms

- \( n \geq 2 \) firms in the market.
- Firm \( i \) has a constant marginal cost of \( c_i \) (not identical across firms) but similar enough.
- Inverse demand function: \( P(Q) = a - Q \), \( Q = q_1 + q_2 + \cdots + q_n \)
- Notation (for simplicity, you can just write out the whole expression):
  \[ q_i = (q_1, q_2, \ldots, q_{i-1}, q_{i+1}, q_n) \]
  \[ \sum_{j \neq i} q_j = q_1 + q_2 + \cdots + q_{i-1} + q_{i+1} + \cdots + q_n \]
  (e.g) If there are 4 firms, then \( q_{-2} \) means \( (q_1, q_3, q_4) \) and \( \sum_{j \neq 2} q_j = q_1 + q_3 + q_4 \)
- Nash Equilibrium?

  \[ \pi_i(q_i, q_{-i}) = \left( a - q_i - \sum_{j \neq i} q_j - c_i \right) q_i \]

  \[ FOC : a - 2q_i - c_i - \sum_{j \neq i} q_j = 0 \]

  \[ BR_i(q_{-i}) = (q_i) = \frac{1}{2} \left( a - c_i - \sum_{j \neq i} q_j \right) \ldots \]

If \( (q_1^*, q_2^*, \ldots, q_n^*) \) is the NE, then this need to satisfy (*) for every firm. However, it’s not easy to solve this using plug-in method as we did so far. Consider the following trick; write out all the best response function at NE.

\[ 2q_1^* = a - c_1 - (q_2^* + q_3^* + \ldots + q_{n-1}^* + q_n^*) \]
\[ 2q_2^* = a - c_2 - (q_1^* + q_3^* + \ldots + q_{n-1}^* + q_n^*) \]
\[ 2q_3^* = a - c_2 - (q_1^* + q_2^* + \ldots + q_{n-1}^* + q_n^*) \]
\[ 2q_{n-1}^* = a - c_{n-1} - (q_1^* + q_2^* + q_3^* + \ldots + q_n^*) \]
\[ 2q_n^* = a - c_n - (q_1^* + q_2^* + q_3^* + \ldots + q_{n-1}^* + q_n^*) \]

By summing these equations over all firms, you’ll get

\[ 2 \sum_{i=1}^{n} q_i^* = na - \sum_{i=1}^{n} c_i - (n-1) \sum_{i=1}^{n} q_i^* \]
\[ \sum_{i=1}^{n} q_i^* = \frac{n}{n+1} a - \frac{1}{n+1} \sum_{i=1}^{n} c_i \]
\[ \sum_{j \neq i} q_j^* = -q_i^* + \frac{n}{n+1} a - \frac{1}{n+1} \sum_{i=1}^{n} c_i \ldots \] (**)
Using (*) and (**),

\[ 2q^*_i = a - c_i - \sum_{j \neq i} q_j \]
\[ = a - c_i - \left( -q^*_i + \frac{n}{n+1}a - \frac{1}{n+1} \sum_{i=1}^{n} c_i \right) \]
\[ q^*_i = a - \frac{n}{n+1}a + \frac{1}{n+1} \sum_{i=1}^{n} c_i - c_i \]
\[ = \frac{1}{n+1} a + \frac{1}{n+1} \sum_{i=1}^{n} c_i + \frac{1}{n+1} c_i - c_i \]
\[ = \frac{1}{n+1} \left( a + \sum_{j \neq i} c_j - nc_i \right) \]

Thus, the \((q_1^*, q_2^*, \cdots, q_n^*)\) satisfying above equation for each firm is the Nash Equilibrium.

- Does this make sense? :
  : If \(n = 2\) and cost are identical \((c_i = c \text{ for all } i)\), then \(q_1^* = q_2^* = \frac{1}{2+1} \left( a + c - 2c \right) = \frac{1}{3} (a - c)\), which is exactly same with what you saw in the lecture note. If too complicated, try to solve for \(n = 3\) or 4 case for yourselves.

### Bertrand Competition with different marginal costs

- Firm 1 has lower marginal cost than firm 2 : \(c_1 < c_2\)
- What is NE and is it unique?
  (The followings are rough description, not considering about the monopoly price. But this is enough at this point.)

1. \(P_1, P_2 > c_2\) and \(P_1 = P_2\) : Both has incentive to cut the price.
\[
\left\{ \begin{array}{l}
  P_1 = P_2 : \text{Both has incentive to cut the price.} \\
  P_1 \neq P_2 : \text{The firm with higher price has incentive to decrease her price.}
\end{array} \right\}
\]

2. \(P_2 < c_2\) and \(P_1 \geq P_2\) : The profit of Firm 2 is negative, so she want to increase her price.

3. \(P_2 < c_2\) and \(P_1 < P_2\) : Firm 1 gets better off by increase her price slightly (to \(P'_1\)) so that \(P_1 < P'_1 < P_2\).

4. \(P_2 > c_2 \geq P_1\) : Firm 1 gets better off by increase her price slightly (to \(P'_1\)) so that \(P_1 < P'_1 < P_2\).

5. \(P_2 = c_2\) and \(P_1 = c_2\) : \(P_1 > c_2\) : Firm 2 will increase her price to some \(P'_2\) to \(c_2 < P'_2 < P_1\)
\[
\left\{ \begin{array}{l}
  P_1 > c_2 : \text{Firm 2 will increase her price to some } P'_2 \text{ to } c_2 < P'_2 < P_1 \\
  P_1 < c_2 : \text{Firm 1 can increase the price closer to } c_2 \\
  P_1 = c_2 : \text{This is the unique NE.}
\end{array} \right\}
\]

- If you just want to show \(P_1 = P_2 = c_2\) is a Nash equilibrium (ignoring the possibility of other equilibria), then you don’t need to exhaust all the cases above. You can just explain why no one will deviate from this strategy.