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Economics 203C (Spring, 2009)
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Problem Set III (due 4/28/09)

1. This exercise continues the investigation of the minimum distance (MD) estimator $\widehat{\theta}_n$ of θ_0, of Problem Sets I and II, where asymptotic normality of the MD estimator $\widehat{\theta}_n$ was derived.

(i) Assume $\widehat{V}_n$ is a consistent estimator of V_0 (the asymptotic covariance matrix of $\sqrt{n}(\widehat{\pi}_n - \pi_0)$). How would you estimate the asymptotic covariance matrix of the MD estimator $\widehat{\theta}_n$?

(ii) Find the optimal choice of the weighting matrix A_n.

2. This exercise continues the Monte Carlo experiments of Problem Sets I and II. (same model and notation). This time we are interested in the finite sample size and power properties of a Wald test of the null hypothesis $H_0 : \beta = 0$ versus the alternative $H_1 : \beta \neq 0$.

(i) Denote the true β in the structural equation $y_i = x_i\beta + \varepsilon_i$ by β_0. For the eight parameter combinations $K = (1, 10)$, $\rho = (.5, .99)$ and $\eta = (.05, 1)$ and for each $\beta_0 \in S := \{-.8, -.6, ..., .6, .8\}$ simulate $R = 1000$ data samples and each time calculate the Wald statistic based on the TSLS estimator. Each time, reject the null hypothesis $H_0 : \beta_0 = 0$ if the Wald statistic is larger than the 5% asymptotic critical value. For each $\beta_0 \in S$ and each of the eight parameter combinations, report the finite sample rejection probabilities. (If $\beta_0 = 0$, you are reporting the rejection probability under the null, if $\beta_0 \neq 0$ you are reporting the *finite sample power* of the test against the particular alternative.)

(ii) Discuss your results. In particular, how do the results vary with K, ρ, and η?

3. Using the same notation as in exercise 2., define

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\begin{aligned}
g_i(\beta) \ &: = (y_i - x_i'\beta)Z_i, \ G_i := (\partial g_i/\partial\beta)(\beta) = -Z_i x_i', \\
\widehat{g}(\beta) \ &: = \sum_{i=1}^{n} g_i(\beta)/n, \ \widehat{\Omega}(\beta) := \sum_{i=1}^{n} g_i(\beta)g_i(\beta)'/n, \\
D(\beta) \ &: = \sum_{i=1}^{n} (\widehat{g}(\beta)'\widehat{\Omega}(\beta)^{-1} g_i(\beta) - 1)G_i/n, \\
LM_{CUE}(\beta) \ &: = n\widehat{g}(\beta)'\widehat{\Omega}(\beta)^{-1}D(\beta)[D(\beta)'\widehat{\Omega}(\beta)^{-1}D(\beta)]^{-1}D(\beta)'\widehat{\Omega}(\beta)^{-1}\widehat{g}(\beta).
\end{aligned}
$$

(i) Show that for $n \to \infty$ we have $LM_{CUE}(\beta_0) \to_d \chi^2_{\dim \beta_0}$, where β_0 is the true structural parameter vector. (Hint: First derive the asymptotic distribution of $n^{1/2}\widehat{g}(\beta_0)$ and then the probability limits of $D(\beta_0)$ and $\widehat{\Omega}(\beta_0)$. You can assume iid observations but not conditional homoskedasticity. State any additional assumptions you use.)

(ii) Redo the Monte Carlo exercise 2., using LM_{CUE} instead of the Wald statistic. Then, compare the performance of the two test statistics.

4. Prove that (i) $O_p(1)o_p(1) = o_p(1)$, (ii) $X_n \to_d X$, for a random variable X, implies $X_n = O_p(1)$, and (iii) discuss what the problem is with a Wald type hypothesis test of $H_0 : h(\theta) = 0$ versus $H_1 : h(\theta) \neq 0$ that rejects at nominal size α when the Wald test statistic W_n is *smaller* than the α quantile $\chi^2_{r,\alpha}$ of a chi-square distribution with r degress of freedom.