UCLA, Department of Economics, Patrik Guggenberger
Economics 203C (Spring, 2009), May 21, 2009

Problem Set VIII=Last PS (due Wed 6/3/09)

1. In class we dicussed the asymptotic result

$$T^{1/2}(\widehat{\pi}_T - \pi_T) \rightarrow_d N(0, \Omega \otimes Q^{-1}) \tag{1}$$

for the conditional MLE estimator $\widehat{\pi}_T$ of the parameters π_T in a VAR(p) model. The estimator converges at convergence speed $T^{1/2}$ to its asymptotic normal distribution. How does this result specialize to the OLS estimator $\widehat{\rho}$ in the case of an AR(1) model

$$y_t = c + \rho y_{t-1} + \varepsilon_t? \tag{2}$$

Which parameters does the limit distribution depend on? What happens when ρ is close to 1? What do you think happens when $\rho = 1$? What do you think the convergence speed is in this case: slower or quicker than $T^{1/2}$? Simulate the finite sample distribution of $T^{1/2}(\widehat{\rho} - 1)$ for $T = 1000$ when $\rho = 1$ and ε_t are iid N(0,1) and $y_0 = c = 0$. Do the same with $T^{\beta}(\widehat{\rho} - 1)$ for another exponent β that you think is suitable.

2. a) In the AR(1) model with no intercept

$$Y_t = \rho Y_{t-1} + u_t, \; u_t = \text{iid}(0, \sigma^2), \; \sigma^2 > 0 \tag{3}$$

show (directly, without quoting any theorem) that there cannot be a stationary causal solution Y_t if $\rho = 1$.

b) For a covariance stationary process Y_t derive a formula [in terms of $\mu := E(Y_t)$, γ_0, γ_1, and γ_2, where γ_k denotes the covariance of Y_t at lag k] for the optimal (in terms of MSE loss) linear forecast of Y_{t+1} based on a constant and Y_{t-1}. Using that result, calculate the optimal linear forecast if Y_t is an AR(1) process given by $Y_t = c + \rho Y_{t-1} + u_t$.

c) If an AR(1) process with intercept is observed every other time period, show that the observed process is still AR(1). Express the parameters of the new process in terms of those of the original process.

3. Consider a VAR(2) model

$$y_t = c + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t \tag{4}$$

with $y_t = (y_{t,1}, y_{t,2})' \in R^2$, $\Phi_1, \Phi_2 \in R^{2x2}$ having coefficients $\Phi_{ij}^{(k)}$ for $i, j, k = 1, 2$, and ε_t $iid(0, \Omega)$. If Φ_1 and Φ_2 are both lower triangular matrices (i.e. $\Phi_{12}^{(k)} = 0$ for $k = 1, 2$) the the optimal one-period-ahead forecast of $y_{t,1}$

$$c_1 + \Phi_{11}^{(1)} y_{t-1,1} + \Phi_{11}^{(2)} y_{t-2,1} \tag{5}$$

depends only on its own lagged values and not on lagged values of $y_{t,2}$ and we say that $y_{t,2}$ does not "Granger cause" $y_{t,1}$.

a) If $\Phi_{12}^{(k)} = 0$ for $k = 1, 2$, explicitly solve the determintal condition ("$|I - \Phi_1 z - \Phi_2 z^2| = 0$ has all roots outside the unit circle") for a causal solution in model (4) as an expression in the coefficients of Φ_1 and Φ_2.

b) How would you test $H_0 : (\Phi_{12}^{(1)}, \Phi_{12}^{(2)})' = 0$ versus $H_1 : (\Phi_{12}^{(1)}, \Phi_{12}^{(2)})' \neq 0$ using a Wald test?

c) Design a Monte Carlo study to investigate the finite sample null rejection probability of the test in part b). How is the null rejection probability affected when in a) there is a root close to or equal to 1?

d) VOLONTARY: Choi, I (2005) "Subsampling vector autoregressive tests of linear constraints", Journal of Econometrics, 124, 55-89 suggests a subsampled Wald test of the null in b). Do you find that subsampling tests have better null rejection probabilities in the simulations in c) than the original Wald test?