Problem Set 8 Pre-Instruction

May 30th, 2009 by Yang

1. Asymptotic Distribution of MLE of AR(1)

See TA note 8 for theoretical derivation of the asymptotic distribution. The data generating process is quite similar to those of Q3 in PS6 and of Q4 in PS7. Let $\rho = 1$. For each r, generate $y_1, \cdots, y_T$ and estimate c and ρ together. Do this $R = 1000$ times then you would get R estimates of $\widehat{\rho}$. Plot the histogram of $T^{1/2}(\widehat{\rho} - 1)$ and compare with that of $T(\widehat{\rho} - 1)$. For finite T, we cannot actually tell which one is similar to the limit distribution. So try the same procedure with different choices of T. You would see that the distribution of $T^{1/2}(\widehat{\rho} - 1)$ shrinks towards 0 as T gets larger, while the distribution of $T(\widehat{\rho} - 1)$ remains as similar figures for different T. You may try to plot $T^{\beta}(\widehat{\rho} - 1)$ for different choices of β.

2. Proof Questions: AR(1)

See TA note 8.

3. Causality Test with VAR(2)

(a) See TA note 8 for clarification of the parameters. Note that

$$|I - \Phi^{(1)}z - \Phi^{(2)}z^2| = \begin{vmatrix} 1 - \Phi_{11}^{(1)}z - \Phi_{11}^{(2)}z^2 & -\Phi_{12}^{(1)}z - \Phi_{12}^{(2)}z^2 \\ -\Phi_{21}^{(1)}z - \Phi_{21}^{(2)}z^2 & 1 - \Phi_{22}^{(1)}z - \Phi_{22}^{(2)}z^2 \end{vmatrix}$$

Use the fact that $\Phi_{12}^{(1)} = \Phi_{12}^{(2)} = 0$ to simplify the determinant. You do not have to explicitly solve for the solutions of the equation, nor to obtain the exact set of $\Phi^{(1)}$ and $\Phi^{(2)}$ that guarantees the causality.

(b) See TA note 8.

(c) The Wald test works quite well when the model is stationary and causal. When there is a unit root for the equation defined in (a), the finite sample distribution of the Wald test does not appoximate χ_2^2 well. If the model is defined noncausal stationary, the above data generating process distorts the result very much, usually resulting in failure of estimation. The example distribution of the Wald statistics is given on the following pages, where the parameters are chosen as, for the causal case,

$$\Phi^{(1)} = \begin{pmatrix} 0.9 & 0 \\ 0.4 & 0.7 \end{pmatrix} \text{ and } \Phi^{(2)} = \begin{pmatrix} -0.2 & 0 \\ 0.7 & -0.1 \end{pmatrix}$$

which has the roots $z = 2, 2, 2.5, 5$, and for the unit root case,

$$\Phi^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \Phi^{(2)} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

FIGURE1. The Asymptotic Distribution of the Estimator of the Unit Root ($R = 1000$)

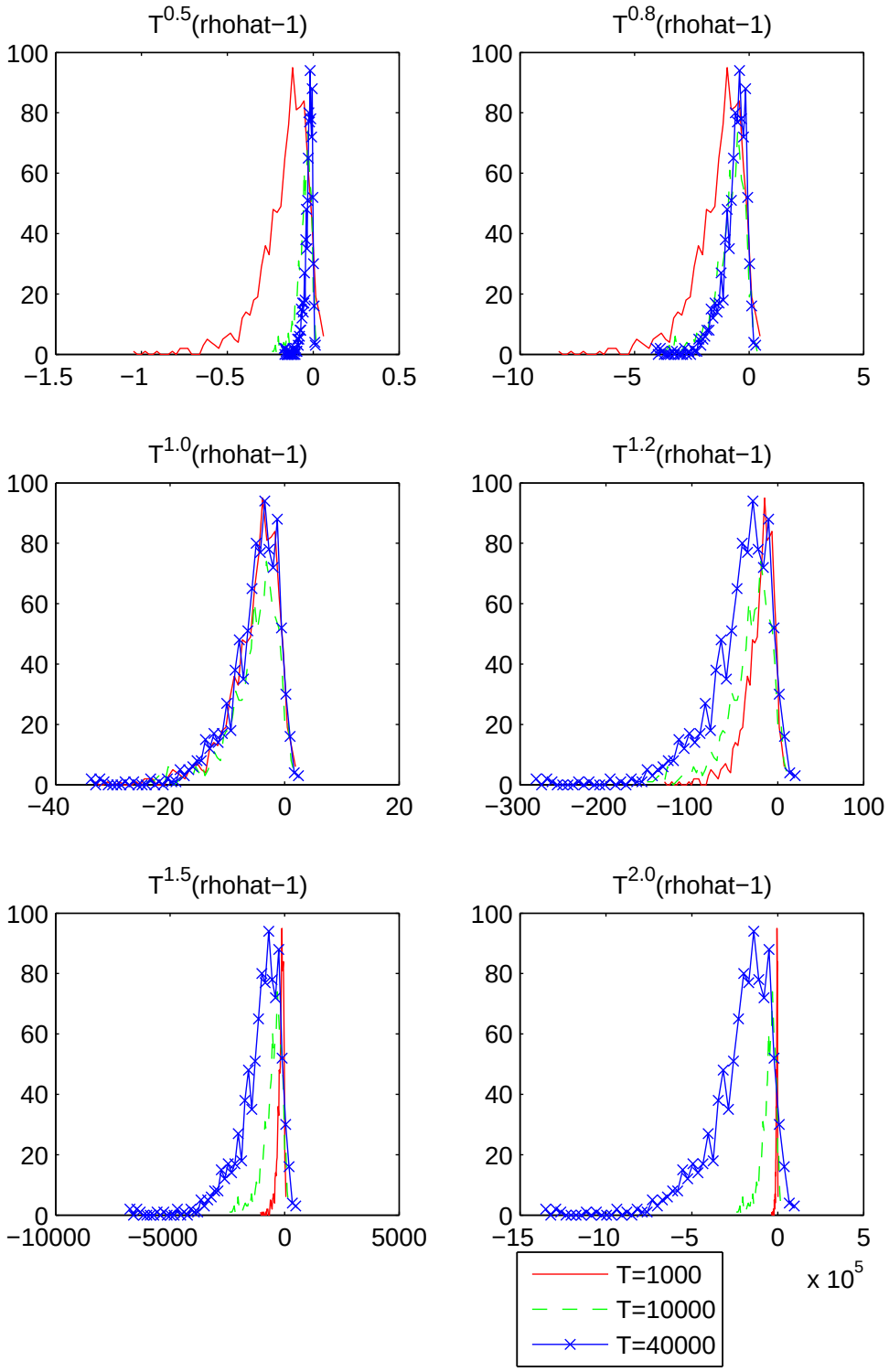

FIGURE2. The Distribution of the Wald Statistic ($T = 100$)

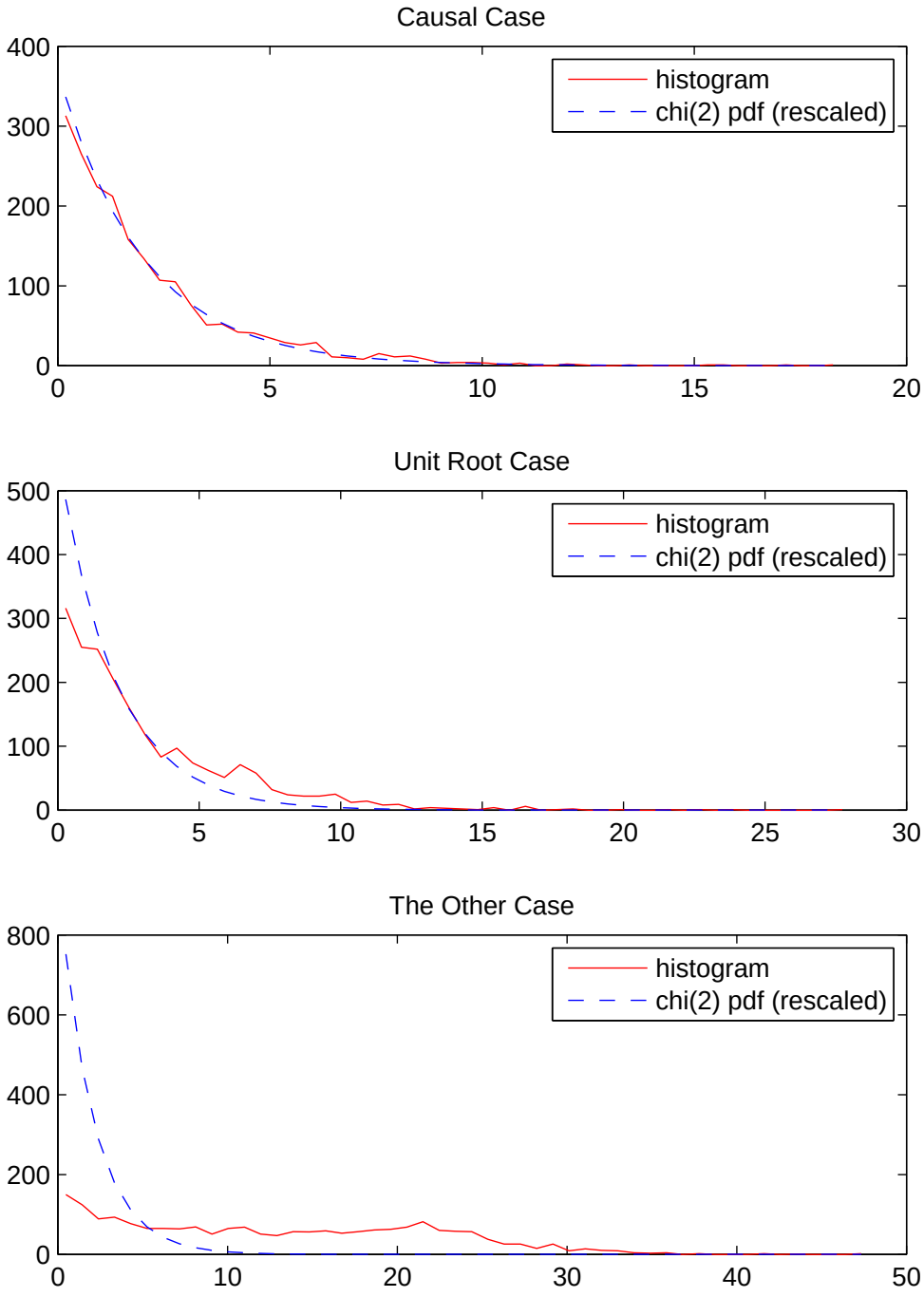

FIGURE3. The Distribution of the Wald Statistic ($T = 1000$)